MAISON
DES EXAMENS
Service Interacadémique
des **Examens & Concours**

BANQUE DE SUJETS

ANGLAIS / MATHÉMATIQUES

SECTION EUROPÉENNE

SESSION 2019

Anglais / Mathématiques

N°	TITRE DU SUJET
D0_52	Core Knowledge
D0_53	Core Knowledge
D0_62	Core Knowledge
D0_65	Core Knowledge
D0_66	Core Knowledge
D0_68	Core Knowledge
D0_73	Core Knowledge
D0_74	Core Knowledge
D0_81	Core Knowledge
D0_82	Core Knowledge
D0_83	Core Knowledge
D0_84	Core Knowledge
D0_91	Core Knowledge
D0_92	Core Knowledge

**MERCI DE VEILLER STRICTEMENT À CE QUE LES CANDIDATS RESTITUENT LEUR
SUJET UNE FOIS L'INTERROGATION ACHEVÉE.**

N°	TITRE DU SUJET
D1_52	Mapping
D1_55	Mapping
D1_62	Mapping
D1_63	Mapping
D1_64	Mapping
D1_65	Mapping
D1_66	Mapping
D1_67	Mapping
D1_68	Mapping
D1_73	Mapping
D1_74	Mapping
D1_81	Mapping
D1_82	Mapping
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D1_91	Mapping

N°	TITRE DU SUJET
D2_71	Differentiation
D2_72	Differentiation
D2_81	Differentiation
D2_82	Differentiation
D2_91	Differentiation

MERCI DE VEILLER STRICTEMENT À CE QUE LES CANDIDATS RESTITUENT LEUR SUJET UNE FOIS L'INTERROGATION ACHEVÉE.

N°	TITRE DU SUJET
D3_52	Sequences
D3_62	Sequences
D3_63	Sequences
D3_65	Sequences
D3_71	Sequences
D3_72	Sequences
D3_81	Sequences
D3_82	Sequences
D3_83	Sequences
D3_91	Sequences

N°	TITRE DU SUJET
D4_51	Statistics
D4_53	Statistics
D4_54	Statistics
D4_55	Statistics
D4_61	Statistics
D4_71	Statistics
D4_72	Statistics
D4_81	Statistics
D4_82	Statistics
D4_83	Statistics
D4_84	Statistics
D4_91	Statistics

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N°	TITRE DU SUJET
D5_52	Advanced Geometry
D5_61	Advanced Geometry
D5_81	Advanced Geometry
D5_82	Advanced Geometry
D5_83	Advanced Geometry
D5_84	Advanced Geometry
D5_85	Advanced Geometry
D5_91	Advanced Geometry
D5_92	Advanced Geometry

N°	TITRE DU SUJET
D7_61	Probability
D7_62	Probability
D7_63	Probability
D7_71	Probability
D7_72	Probability
D7_73	Probability
D7_91	Probability
D7_92	Probability
D7_93	Probability
D7_94	Probability

MERCI DE VEILLER STRICTEMENT À CE QUE LES CANDIDATS RESTITUENT LEUR SUJET UNE FOIS L'INTERROGATION ACHEVÉE.

BACCALAURÉATS GÉNÉRAL ET TECHNOLOGIQUE
SESSION 2019

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »
Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

CORE KNOWLEDGE
Sujet D0 - 52

Recap:
Imperial units: 1 foot is roughly 30 cm, 1 gallon roughly 4.5 litres.
Volume of a cylinder: $\pi \times r^2 \times h$, with r the radius of the base circle and h the height of the cylinder.

Exercise:
The paintwork has to be redone in Debbie's bedroom.

His rectangular bedroom is 3 m wide, 4 m long and its walls are $2\frac{1}{2}$ m high.

1) At first, Debbie wants to know the surface area of all the walls, door and window included. Help her.

2) Knowing that the door is 3 feet wide and 7 feet high, and knowing that the window is a square whose side measures 6 feet, calculate the surface area that Debbie has to repaint.

3) At the store, Debbie discovers that 1litre of paint is necessary to cover a surface area of 2.5 m². How much paint does she need?

4) Debbie has the choice between two different kinds of paint pots: one which contains 3 litres and costs £18, and one which contains 1 gallon and costs £20.
    Which is the more economical?

5) Finally, she decides to choose the one-gallon paint pots. These are cylindrical. Their height is 30 cm.
    Debbie wants to transport the pots in a box whose dimensions are 15 cm × 45 cm × 30 cm (width × length × height). Can she?

CORE KNOWLEDGE
Sujet D0 – 53

Recap: We can estimate the median from the graph by reading off the half-way value on the vertical axis.
The lower quartile (LQ) is the value one-quarter of the way into the distribution.
The upper quartile (UQ) is the value three-quarters of the way into the distribution.
In a histogram, the frequency of each class is represented by the area of the corresponding bar.

Exercise: The time spent by a class of 26 pupils revising a Math test has been collected in the following table. The graph of the corresponding cumulative frequency curve is also shown.

Time spent (in minutes)	Number of pupils
Less than 10	2
10-20	4
20-25	4
25-30	6
30-40	6
40-60	4
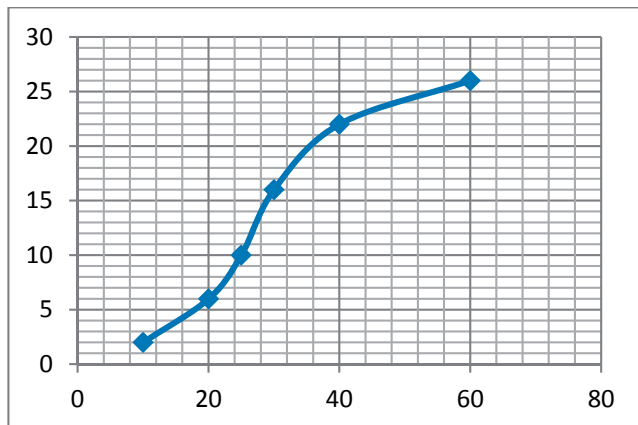
1)
    a. Calculate the mean time spent revising the test by this class.
    b. Estimate graphically the number of pupils who revised between 35 and 55 minutes.
    c. Estimate graphically the median time, the lower quartile and the upper quartile.
       Comment on your answers.
    d. Calculate the interquartile range. Comment on your answer.

2) The teacher would like to draw a histogram to show the data using the following scale:
-   1 cm to represent 5 minutes on the x-axis
-   1 cm^2 to represent 1 pupil

She drew a table to show the heights of the bars of the different classes.

Number of minutes	0-10	10-20	20-25	25-30	30-40	40-60
Height of the bar (cm)	x	y	z	t	u	1

    a. Explain why the height of the bar of the last class is 1 cm.
    b. Find x, y, z, t and u.

CORE KNOWLEDGE

Sujet D0 - 62

Exercise: The Tealicious company owns jumbo hopper barges and uses them to ship cubic containers on the Thames.

Part A: each cubic container has a side of 5 ft.

1. What is the volume of a cubic container?
2. A jumbo hopper barge is a huge boat that can be considered as a rectangular prism. It is 100 feet long, 40 feet wide, and 12 feet deep.
    a. Prove that 384 cubic containers are needed to match the same volume as a jumbo hopper barge.
    b. Can 384 cubic containers fit in a jumbo hopper barge? Why? Show that the greatest number of cubic containers that can be loaded in a jumbo hopper barge is 320.

Part B: each cubic container can contain either black tea or green tea and can be shipped either to London or to Oxford. The Tealicious company warehouse contains 6,200 cubic containers, 1,860 of which containing black tea. 20% of the black tea containers should be sent to Oxford whereas 60% of the green tea containers should be sent to London. A cubic container is chosen at random in the warehouse.

Let L be the event "the cubic container is sent to London" and G the event " the cubic container contains green tea".

1. Show the data in a tree diagram and describe it.
2. Show that the probability of event L is 0.66.
3. Deduce the number of cubic containers shipped to London.
4. How many jumbo barges are needed for that shipping?

BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
SESSION 2019

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »
Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

CORE KNOWLEDGE

Sujet D0 - 65

Recap: 1 litre = 1 dm^3 ; 1 mile = 1.6 km; 1 gallon = 4.5 litres

Mr. Thomas has to carry a great quantity of cardboard boxes from his house to his daughter's in his trunk.
1) He has to carry 28 boxes of the same size: each is 15 cm wide, 20 cm long and 25 cm high. However, the capacity of his trunk is 200 litres. Could he carry all of them in one trip or will he need to make two trips?
2) For one trip, he has to travel 12 km. The fuel consumption of his car (an Aston Martin) is about 19.8 mpg (miles per gallon). How much petrol will Mr. Thomas need for one journey there and back?
3) The outward journey takes him 15 minutes. Find his average speed in miles per hour.
4) There are two sets of traffic lights on Mr. Thomas's route to his daughter's. If the traffic light is green, he can drive, otherwise, he has to stop.
The probability that he gets stopped at the first set is 0.3.
If he doesn't stop at the first set, then the probability that he has to stop at the second set is 0.4.
On the other hand, if he gets stopped at the 1st set, the probability he has to stop at the 2nd set is 0.8. Explain how you will draw a tree diagram to show this probability.
What is the probability that he drives past two green lights?

CORE KNOWLEDGE

Sujet D0 - 66

EXERCISE

Pythagoras' theorem states that in any right-angled triangle: $a^2 + b^2 = c^2$, where *c* represents the length of the hypotenuse, and *a* and *b* represent the lengths of the other two sides (called legs of the triangle).

A Pythagorean triple is any set of positive integers that satisfy the relationship $a^2 + b^2 = c^2$. Famous Pythagorean triples include (3; 4; 5) and (5; 12; 13).

1. (*i*) Show that (8; 15; 17) is a Pythagorean triple.
   (*ii*) Could you find *a* and *b* such that (*a*; *b*; 10) is a Pythagorean triple?

2. (*i*) If you add up the same positive integer to each term of a Pythagorean triple, do you still have a Pythagorean triple?
   (*ii*) If you multiply each member of a Pythagorean triple by the same positive integer, do you still have a Pythagorean triple?
   (*iii*) How many different Pythagorean triples do you think there are?

3. The seventeenth-century French mathematician Pierre de Fermat stated without a single proof that the equation $x^n + y^n = z^n$ has no whole number solutions for *n* greater than 2. What became "Fermat's last theorem" baffled the greatest minds on the planet for over three centuries: it has only been proved recently by a British mathematician named Andrew Wiles.

   A rational number can be written under the form $\dfrac{a}{b}$, where *a* and *b* are integers, $b \neq 0$.

   We know now for sure that it is impossible to find *x, y, z,* positive **integers** such that $x^3 + y^3 = z^3$ for instance. But do you think you could find *x, y, z,* positive **rational numbers** such that $x^3 + y^3 = z^3$?

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »
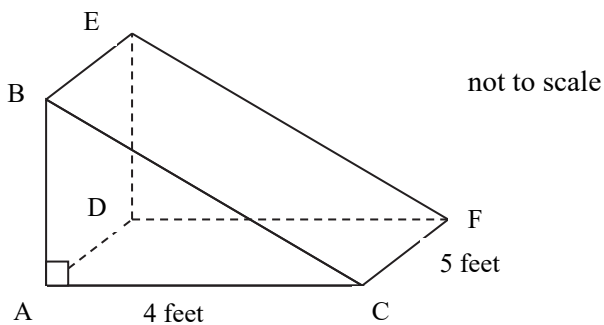Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

CORE KNOWLEDGE
Sujet D0 - 68

You may use the following formula: Volume of prism = base area × length.

EXERCISE:

1. ABCDEF is a right-angled prism.
One side length of its base is 4 feet and its height is 5 feet. Its volume is 35 ft².
   a) Calculate the length of AB.
   b) Calculate the length of BC. Give your answer to the nearest tenth.

not to scale

2. Adam is twice as old as Jane.
   Charlie is three years younger than Jane.
   The sum of all their ages is 53.
    How old is Jane?

3. First three diagrams in a sequence are shown.
   How many squares will there be in a diagram 120?
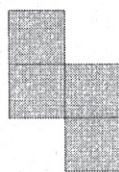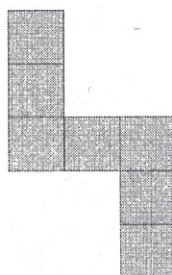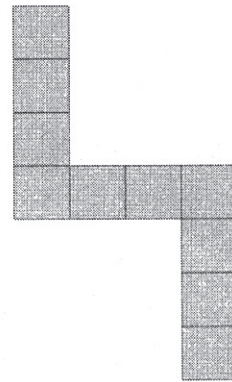

Diagram 1   Diagram 2   Diagram 3

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »
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CORE KNOWLEDGE
Sujet D0_73

Recap:

Two numbers a and b are in the ratio p:q if and only if $\dfrac{a}{p} = \dfrac{b}{q}$ (with $p \neq 0 \; and \; q \neq 0$)

Volume of a cylinder: $\pi \times r^2 \times h$, with r the radius of the base circle and h the height of the cylinder.

Volume of a prism: area of the base × height of the prism.

Exercise:

John is a great cook and he wants to cook apple jam.
In the recipe, the weight of apples and sugar is in the ratio 3:2.

1) If he wants to get 1 kg of jam, what are the amounts of apples and sugar he needs?

2) John's sister gave him a 1.5kg pack of apples. How much sugar does he need if he wants to use all the apples for his jam?

3) John has got two types of pots to put his jam in.
   The first one is a cylinder and the second one has the shape of a cuboid whose base is a square. Each pot is 15 cm high.
   The base circle of the cylinder has a diameter of 5cm whereas each side of the square is 4.5 cm.
   John wants to use the pot which contains the most.
   Can you help him?
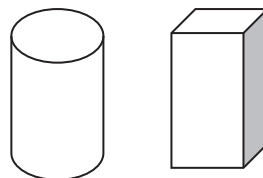
4) John has two children, Ben and Ava. They want to taste his jam. Ben, the elder child, begins, but as he is fussy about his food, the probability he likes it is only 0.6.
   Moreover, Ava, the younger child, is easily influenced.
   If Ben likes the jam, the probability that Ava likes it is 0.8. Otherwise the probability that Ava likes it is 0.5.
   Draw a tree diagram and find the probability that Ava likes her dad's jam.

5) Last year, John cooked several types of jam. He got 15 pots of orange jam (200g each), 10 pots of pear jam (150g each) and 12 pots of lemon curd (100g each). What is the mean mass of a pot he cooked last year?

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »
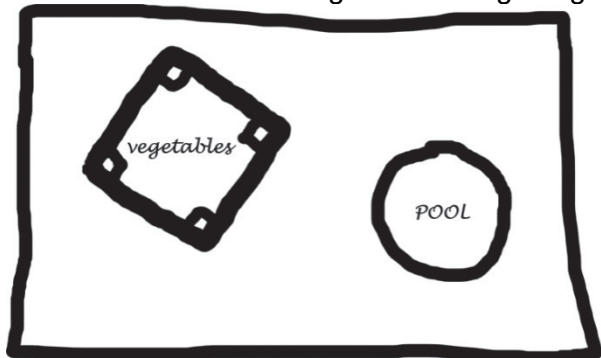Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

CORE KNOWLEDGE
Sujet D0_74

Given the radius R of a circle, the area of a circle can be calculated using the formula: πR^2

Jonathan wants to redesign his rectangular garden. He made a freehand drawing:

In the garden, there will be a square space for vegetables, and a round swimming-pool. Jonathan wants to plant grass everywhere else.

The diameter of the pool is 3.5 meters
Each side of the "vegetables square" is 5 meter long.

The garden is 34 meter long and 15 meter wide.

In his favorite shop, he saw these ads :

SPECIAL OFFER
-10%
DISCOUNT WILL BE APPLIED AT CHECKOUT
POOL KIT
(RECTANGULAR or ROUND)
$474.99

QUALITY GRASS SEEDS

the bag of 1 lb : $8.95
Add to cart

the bag of 5 lbs : $32.50
Add to cart

1 lb covers up to 30m²

1) Find out how much Jonathan should spend to redesign his garden (for π use 3.14. All results should be rounded to 2d.p.).
Be ready to describe each step of your reasoning.

2) Gary's garden is a rectangle as well. Its perimeter P is the same as Jonathan's.
     a) Find a value for P.
     b) Do both gardens have the same area?

CORE KNOWLEDGE
Sujet n° D0_81

Recap:

Two numbers a and b are in the ratio p:q if and only if $\frac{a}{p} = \frac{b}{q}$ (with $p \neq 0 \ and \ q \neq 0$)

Volume of a cylinder: $\pi \times r^2 \times h$, with r the radius of the base circle and h the height of the cylinder.

Volume of a prism: area of the base × height of the prism.

A cuboid is a prism whose base is a rectangle.

Exercise:

Sandy loves making cupcakes and her friends know it. She owns different cake tins and uses them according to her needs.

1) Volumes of the different cake tins:

   a) For Christmas, the cupcakes cooked by Sandy are cylinders. The cake tins have a base diameter of 5cm and they are 4 cm high. What's the volume of each tin?

   b) For a dinner, Sandy uses cuboid tins. They are 4 cm high and their base is a rectangle, 5 cm long and 3 cm wide. What's the volume of this tin?

   c) For a child's birthday, she prefers cooking cupcakes with special tins: triangle based prisms. The base is an isosceles right-angled triangle. Its hypotenuse is $\sqrt{50}$ cm.

      Work out the length of the sides of this triangle, then the volume of this tin given that it is 4 cm high.

2) Ratio cake:cream

Of course, at the top of each cupcake, Sandy adds a lovely hat of cream. She estimates that a cupcake is perfect if the ratio of the volumes of cake to cream is 5:3.

For a 60 cm³ cake, calculate the corresponding volume of cream.

3) A particular dinner:

To celebrate her friend's A-Levels, Sandy decides to organize a dinner. She wants to bake 20 cupcakes. Knowing that she needs 2 eggs for a volume of 96 cm³ (cake and cream altogether), how many eggs does she need?

CORE KNOWLEDGE
Sujet D0_82

Unit conversions:	Abbreviations :
1 foot = 30.48 centimetres	1 foot = 1 ft.
1 foot = 12 inches	1 inch = 1'' or 1 in.
Area of a disc (given a radius r): $\pi\,r^2$	

Darts is the sport in which small missiles, called "darts", are thrown at a circular dartboard fixed to a wall.
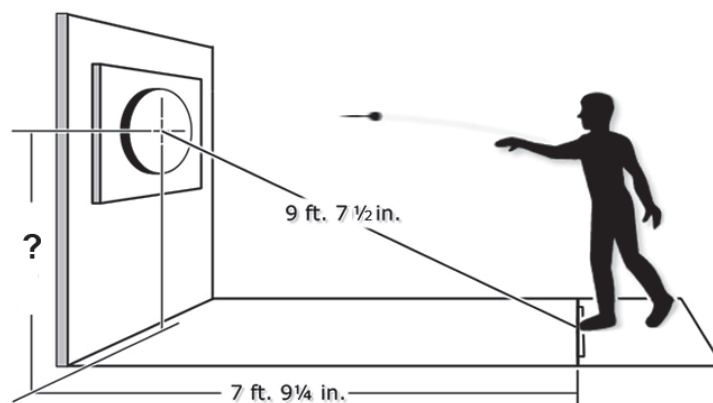

Dartboard   Darts

The throw line, called *oche* (say "oki"), is the line behind which the throwing player must stand. The centre of the dartboard is called *centre bull* or *bull's eye* or *bull*. It scores 50 points.

The distance from the centre of the bull to the oche is 9 feet and 7 ½ inches. The distance between the wall and the oche is 7 feet and 9 ¼ inches.
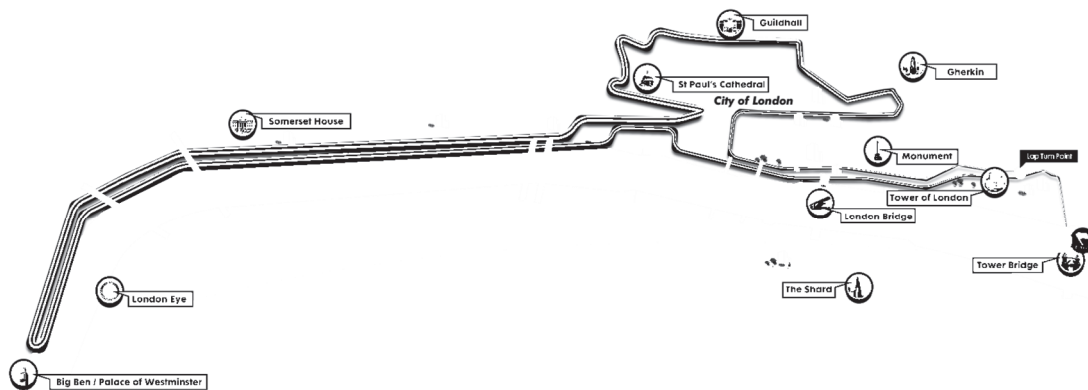

9 ft. 7 ½ in.

?

7 ft. 9¼ in.

1) a) Convert 1 inch into centimetres.
   b) Convert the given distances into centimetres (round to one decimal place).
2) a) Given that the wall is perpendicular to the floor, work out the distance between the floor and the center of the bull (round to the nearest cm).
   b) The real distance is 172.72 cm. Convert it into feet and inches.
3) A player randomly throws a dart (and the dart hits the dartboard). The diameter of the dartboard is 34 cm. The diameter of the bull is 1.4 cm. What is the probability that the dart hits the bull? (to 4 dp)

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Académies de Paris-Créteil-Versailles
Binôme : Anglais / Mathématiques

CORE KNOWLEDGE

Sujet D0_83

Document 1 : London 2017 World Athletics Championships

IAAF World Championships Marathon, Sunday 6 August 2017
For course details & Championships info visit london2017athletics.com

Set against some of the capital's most historically-significant and picturesque backdrops, the 26.2 mile marathon route will start and finish at Tower Bridge in central London. The route will comprise four laps of a 10 km course on closed roads, heading west along Victoria Embankment towards the Houses of Parliament, then back alongside the River Thames to St Paul's Cathedral, and returning to the Tower of London.

Source : http://www.london2017athletics.com

Questions:

1. Present 2017 IAAF world championships marathon: total length of the course, length of one lap, length between the start and finish points and the lap turn point. Give all your answers in miles and in kilometers.
2. During the 26.2 mile men's marathon (rounded to the nearest tenth) at the 2012 Summer Olympics:
    *   85 runners finished the race. The winner in 2:08:01 and the last runner in 2:55:54.
    *   20 other runners did not finish the race.
    Give an interval for the total amount of miles run by all the participants together. Present your results in standard form, to 2 significant figures.
    If all the 85 participants who finished the race had done it one after the other, how many days would it have taken?

Notes:
Backdrop = *décor, arrière-plan*
1 mile = 1609,34 meters
IAAF = International Association of Athletics Federations

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »
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Binôme : Anglais / Mathématiques

CORE KNOWLEDGE
Sujet n° D0_84

Three, One, Four, One, Five, and On
The numbers recount their endless tale.
Three - Barefoot green, a silent voice.
White as hunger, One is twice
Bright like babies' eyes.
Four is timid, envious of E.
Five, Punctuation or a pregnant sigh
Precedes proud Nine, colour of falling night.
Two, an unfastened knot,
A wayward wind, the hollow of Six resounding.
Nearby, Eight, a cloud of fireflies above a lake
Over which I skim Sevens
Remembering that Zero is nothing but a circle.   *Pi Poem*, Daniel TAMMET, 2009

Daniel Tammet (born January 31,1979) is an English writer, essayist, translator, and autistic scientist. He holds the European record for reciting π from memory to 22,514 digits in five hours and nine minutes on March 14th 2004.

1.
   a) Use your calculator to work out   π-3.14   and   22/7-π   correct to 5 dp.
   b) Daniel Tammet chose to recite π on Pi Day: March 14th (**3**rd month **14**th day).
   Why do you think some mathematicians believe Pi Day should be July 22nd?

2.
   a) To this day more than 22 trillion digits of π have been discovered. An average person can read out approximately 120 digits/min.
   Keeping this pace, how long would it take to recite these digits?
   b) Assuming a total world population of roughly 7 billion people, how many digits of π would everyone have to memorize in order to preserve all known digits of π?

3. Let's view π as a big, random string of numbers. The odds of finding a string of digits in the first 100 million digits of π are:

String length	1-5	6	7	8	9	10	11
Chances of finding	100 %	nearly 100 %	99.995 %	63 %	9.5 %	0.995 %	0.09995 %

   a) If we search for the digit "6" in π, what is the chance that a digit picked at random in the first 100 million decimals of π is equal to "6"?
   b) If we search for the string of digits "61" in π, what is the chance that a string of two digits picked at random in the first 100 million decimals of π is equal to "61"?

BACCALAURÉATS GÉNÉRAL ET TECHNOLOGIQUE

SESSION 2019

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »

Académies de Paris – Créteil – Versailles

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CORE KNOWLEDGE

Sujet D0_91

<u>Sapphires and rubies in the sky</u>

" 21 light-years away from us in the constellation Cassiopeia, a planet orbits its star with a year that is just three days long. Its name is HD219134b. With a mass almost five times that of Earth it is a so-called "super-Earth". Unlike the Earth however, it most likely does not have a massive core of iron, but is rich in calcium and aluminum. "Perhaps it shimmers red to blue like rubies and sapphires, because these gemstones are aluminum oxides which are common on the exoplanet" says Caroline Dorn, astrophysicist at the Institute for Computational Science (University of Zurich). "

Source : eurekalert.org   19-dec-2018

|	
Earth mass : 5.9722×10^{24} kg (standard form) Earth diameter : 12,742 km Light speed : 299,792,458 m.s^{-1} Volume of a sphere : $\dfrac{4\pi R^3}{3}$	Density = $\dfrac{mass\ (in\ kg)}{volume\ (in\ m^3)}$ Temperature conversions : $[°C] = [K] - 273.15$ $[°C] = ([°F] - 32) \times \dfrac{5}{9}$

For easier reading, "planet HD219134b" may be shortened into "planet HD".
s.f. means significant figures.

1)	Work out an estimate for the mass of planet HD (standard form, 1 s.f.).
2)	a) Given that *"a light-year is the distance that light travels in vacuum in one Julian year (365.25 days)"*, convert a light-year into kilometers (standard form, 2 s.f.).

	b) Work out the distance between Earth and planet HD (1 s.f.).

3)	Work out the volume of Earth (2 s.f.).
4)	Compute the density of Earth (3 s.f.).
5)	

	The mantle's mass represents 65% of Earth's mass, while the core represents 33%.
	Work out the crust's mass (2 s.f.)
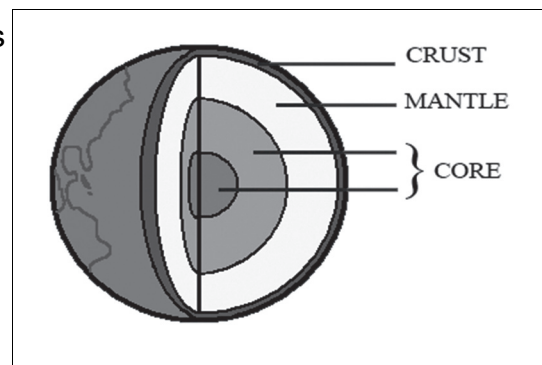
6)	HD's equilibrium temperature is around 1015 Kelvin. Convert this temperature into Fahrenheit.

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CORE KNOWLEDGE

SUJET D0_92

6174 has been known as Kaprekar's constant since Indian mathematician Kapreka discovered it in 1949. This number is notable for the following rule:

A. *Take any four-digit number, using at least two different digits. (Leading zeros are allowed.)*
B. *Arrange the digits in descending and then in ascending order to get two four-digit numbers, adding leading zeros if necessary.*
C. *Subtract the smaller number from the bigger number.*
D. *Go back to step B and repeat.*

1) Apply this algorithm to 6174.

2) Apply this algorithm to two other 4-digit numbers of your choice.
    a) What do you notice?
    b) How many steps did you use?

3) Is there such a 3-digit number? Explain your procedure.

4) Is there such a 2-digit number? Explain your procedure.

A Harshad number is a number divisible by the sum of its digits

A happy number is defined by the following process : starting with any positive integer, replace the number by the sum of the squares of its digits and repeat the process until it reaches 1. If it does not, then it is an unhappy number.

5) Is 6174 a Harshad number?

6) Is 6174 a happy number?

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »
Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

MAPPING
Sujet D1 – 52

Recap:
A quadratic function f of x is a function in the form $f(x) = ax^2 + bx + c$ with a, b, c three real numbers and $a \neq 0$.
The graph of this function is a parabola, a line symmetric curve,
and the x-coordinate of its vertex is $x = -\frac{b}{2a}$.

Romeo and Juliet
It's May 4th 2015 at night. Juliet is in her bedroom with the window half-open. Romeo wants to leave her a message but his cell phone is out of order. He writes his message on a piece of paper and wraps it around a stone that he tries to throw through her window.
Juliet's window is at a height of 3.80m and it is 1 m high.
It is a sash-window[1] and its half-top is closed whereas its half-bottom is open.
Romeo is 3 m away from Juliet's wall.
Romeo throws the stone. The height h of the stone is a quadratic function of x, x being the horizontal distance from Romeo after beginning the throw.
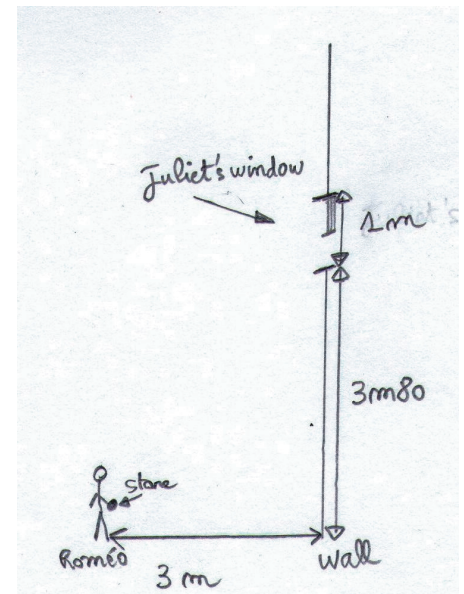At the beginning of the throw, the stone is 1 m high.
Then, when $x = 1$, the stone is 6 m high,
and when $x = 2$, the stone is 7 m high.

1) Give three values of (x, h).

2) Prove that $h(x) = -2x^2 + 7x + 1$.

3) What could you say about the trajectory of the stone?

4) Do you think that the stone with the message will reach Juliet?

   Can it reach Juliet without her window being broken?

[1] sash-window: *fenêtre à guillotine*

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MAPPING

Sujet D1 – 55

Quadratics: the roots of the quadratic equation $a\,x^2 + b\,x + c = 0$, where, $a \neq 0$, are given by the formula $x = \dfrac{-b \pm \sqrt{\Delta}}{2\,a}$, if the discriminant $\Delta = b^2 - 4\,ac$ is a positive real number.

Formulas: area of a circle with a radius of r : $\pi \times r^2$

Ratios: two numbers a and b are in the ratio n **:** m if $\dfrac{a}{n} = \dfrac{b}{m}$, where n and m are two integers different from 0.

Pizzas often come in three sizes: large, medium and individual. When you change the size of the pizza, how does the amount of cheesy, delicious topping compare to the amount of crust…

1) Calculate the total area for each pizza: large, medium and individual. Then, calculate how much of the total area is the cheesy, delicious topping part, and how much is crust. Which size would you prefer and why?

Radius	7 inches	6 inches	3 inches
Crust Thickness	1-inch	1-inch	1-inch
Total area			
Inside area			
Crust area			

2) Most pizzas have a 1-inch wide crust. Based on this

    a) Write functions for the total area, the inside area, and the crust area in terms of the radius, r.

    b) For which size pizza is the inside area equal to the crust area… and would you <u>order</u> it?

3) a) Calculate the ratio of crust area to pizza area for the 7-inch radius pizza.

    b) Which size of pizza should you order if you want the ratio of crust area to pizza area to be 1 **:** 3?

<u>Order</u> = commander

Binôme : Anglais / Mathématiques

MAPPING
D1- 62

The first part of this page is a summary that can help you do the exercise.

▸ Quadratic functions are polynomials in which the largest exponent is 2.
The graph is always a parabola, and the general form of the equation is $y = ax^2 + bx + c$.
If $a > 0$, the parabola opens up and has a minimum value.
If $a < 0$, the parabola opens down and has a maximum value.
The x-coordinate of the vertex of the parabola is equal to $-\frac{b}{2a}$, and the line of symmetry is the vertical line whose equation is $x = -\frac{b}{2a}$.

▸ When $b^2 - 4ac > 0$, the solutions of the quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0$) are given by the quadratic formula : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
If $b^2 - 4ac = 0$, the quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0$) has only one solution : $x = -\frac{b}{2a}$.
If $b^2 - 4ac < 0$, the quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0$) has no real solution.

The Fish&Ship Company wants to ship containers of cod fish to London. The cost, in pounds (£), to ship x containers is a quadratic function of x that is to say that $C(x) = ax^2 + bx + c$, where a, b and c are real numbers and $a \neq 0$. The fixed costs of the company are £ 2750; the cost to ship 10 containers is £4250 whereas the cost to ship 20 containers is £8550.

   1- Show that for all real number $x, C(x) = 14x^2 + 10x + 2750$.

The revenue for each container transported is £570.

Let $P(x)$ be the profit of the company for x containers shipped.

   2- Show that for all real number x, $P(x) = -14x^2 + 560x - 2750$.
   3- How many containers at least must be shipped to have a positive profit?
   4- What can you advise the manager of the Fish&Ship Company maximize its profit?
   5- Describe the graph of P in a cartesian coordinate plane.

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MAPPING
Sujet D1 - 63

<u>Exercise:</u>

We know that the roots of the quadratic equation $x^2 + b\,x + c = 0$, are given by the formula $x_1 = \frac{-b + \sqrt{\Delta}}{2}$, and $x_2 = \frac{-b - \sqrt{\Delta}}{2}$ if the discriminant $\Delta = b^2 - 4c$ is a positive real number.

We suppose the discriminant is a positive number.

1. Use the quadratic formula to show that $x_1 + x_2 = -b$ and $x_1 x_2 = c$.

2. We want to solve $x^2 + 3x - 28 = 0$ without using the quadratic formula.
   a. Find all pairs of integers x_1 and x_2 such that $x_1 x_2 = -28$.

   b. In the previous pairs find which pair x_1 and x_2 is such that $x_1 + x_2 = -3$.

   c. What are the solutions of $x^2 + 3x - 28 = 0$?

   d. Solve $x^2 - 17x + 72 = 0$ with the same method.

   e. Solve $x^2 + 13x + 36 = 0$ with the same method.

3. Let x and y be two numbers such that their product is -15 and their sum is 2.

   a. Show that x must be a solution of a quadratic equation.

   b. Solve the quadratic equation with the quadratic formula.

   c. Find out the values of x and y.

MAPPING
Sujet D1 - 64
The first part of this page is a summary that can help you do the exercise.

▸ Quadratic functions are polynomials in which the largest exponent is 2.
The graph is always a parabola, and the general form of the equation is $y = ax^2 + bx + c$.
If $a > 0$, the parabola opens up and has a minimum value.
If $a < 0$, the parabola opens down and has a maximum value.
The x-coordinate of the vertex of the parabola is equal to $-\frac{b}{2a}$, and the line of symmetry is the vertical line whose equation is $x = -\frac{b}{2a}$.

▸ When $b^2 - 4ac > 0$, the solutions of the quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0$) are given by the quadratic formula : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
If $b^2 - 4ac = 0$, the quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0$) has only one solution : $x = -\frac{b}{2a}$.
If $b^2 - 4ac < 0$, the quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0$) has no real solution.

A factory produces umbrellas. It can produce up to 300 umbrellas per week.
The cost to produce umbrellas depends on the number of umbrellas produced. If we label it x, the director has to pay a cost in £ equal to $C(x) = -0.10x^2 + 30x + 1\,200$.
Each umbrella is sold £18.
   1) If the factory produces 150 umbrellas, how much does it cost? Does the factory make a profit in that case?

For the following questions, we label x the number of umbrellas produced.
   2) We already know that the expenses are given by the value of $C(x)$. What kind of function is C? Explain how you can draw the curve of function C.
   For how many umbrellas produced does the factory have the highest expense?
   3) Express the receipts, R, in terms of x. What kind of function is R? Explain how you can draw the curve of function R.
   4) Express the profit P in terms of x.
   5) For how many umbrellas produced does the factory earn money?
   6) Explain how you can find the highest profit per week and give its value.

MAPPING

Sujet D1 - 65

The first part of this page is a summary that can help you do the exercise.

▸ Quadratic functions are polynomials in which the largest exponent is 2.
The graph is always a parabola, and the general form of the equation is $y = ax^2 + bx + c$.
If $a > 0$, the parabola opens up and has a minimum value.
If $a < 0$, the parabola opens down and has a maximum value.
The x-coordinate of the vertex of the parabola is equal to $-\frac{b}{2a}$, and the line of symmetry is the vertical line whose equation is $x = -\frac{b}{2a}$.

▸ When $b^2 - 4ac > 0$, the solutions of the quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0$) are given by the quadratic formula : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
If $b^2 - 4ac = 0$, the quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0$) has only one solution : $x = -\frac{b}{2a}$.
If $b^2 - 4ac < 0$, the quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0$) has no real solution.

EXERCISE

The height (in metres, from the ground) of a stone launched from a catapult is given by:

$$h(t) = 20t - 9.8t^2$$

where t is the time (in seconds) after the moment of launching.

When the question is about a time, give your answer correct to 1 millisecond (= correct to 3 d.p.).

1) Find when the stone hits the ground.

2) For how long is the stone higher than 5 metres above the ground?

3) Could the stone reach a height of 12 metres above the ground?

4) Explain how you can find the maximum height of the stone.

5) Describe the graph of function h.

Source: The Centre for Innovation in Mathematics Teaching (CIMT)

MAPPING
Sujet D1- 66

The first part of this page is a summary that can be useful to do the exercise.

A linear function is a function f which can be written in the form $f(x) = ax + b$.
The graph of a linear function is a straight line.

The overheads are also called burden costs, they correspond to the expenses that cannot be avoided.

Aicha is the manager of a company which manufactures calculators. She prepares a report on production costs, expenses and returns.
Each calculator costs the company £15 to produce. In addition, the company has monthly overhead costs of £19,710.
The selling price of each calculator is £45.

1. Show that the total cost, C, of producing x calculators each month, is:
$$C(x) = 15x + 19710.$$

2. Write a formula describing the selling price, $S(x)$, of x calculators.

3. Plot and label the graphs of functions C and S, for $0 \leq x \leq 1000$.

4. The point of intersection of the two graphs is called the *break-even point*.
Explain what this means in terms of the given problem.
Find the coordinates of the break-even point (point of intersection).

5. Shade the portion between the two graphs to the left of the break-even point.
Explain what this portion represents.

Profit may be defined as the selling price minus the total cost.

6. Write a formula describing the profit obtained, P, after selling x calculators.

7. Determine whether a profit or loss is made when:
    a. 400 calculators are sold in a particular month;
    b. 800 calculators are sold in a particular month.
        Check your answer with the graph you drew in question 3.

MAPPING

Sujet D1 - 67

Summary:

A quadratic polynomial is a polynomial of the form $ax^2 + bx + c$. The values of *x* for which a quadratic is equal to zero are called the roots of this quadratic. The number of real roots of a quadratic depends on the sign of $\Delta = b^2 - 4ac$ which is called the discriminant.

-        If $\Delta > 0$ then the quadratic has two real roots.
-        If $\Delta = 0$ then the quadratic has one real root (two equal real roots).
-        If $\Delta < 0$ then the quadratic has no real root.

The extremum of a quadratic occurs when *x* is equal to $-\dfrac{b}{2a}$.

Exercise :

A ball dropped with no initial velocity falls free.

With a chronophotography, we know how far the ball has fallen down every 20 ms (milliseconds).

We write *d*(*t*) the distance in cm the ball has fallen down *t ms* after it was dropped.

t	0	20	40	60	80	100	120
d(*t*)	0	0,2	0,8	1,8	3,2	5	7,2

1. Represent (*t*, *d*(*t*)) in a coordinate system.

2.
        a. Is *d* a linear function?
        b. Can you make any conjecture about the nature of *d*?

3. We want to find *a*, *b*, *c* such as *d*(*t*) = *a t*² + *bt* + *c*.
   Use the table above to find the values of *a*, *b* and *c*.

4. Does the expression (found in question 3) work for all the values of the table?

5. We suppose the expression of *d*(*t*) found in question 3 is valid for any value of *t*.
   The ball was dropped from a height of 1 m above the ground. When does it reach the floor?
   Give the time in seconds.

MAPPING

Sujet D1 - 68

Recap

A quadratic polynomial is a polynomial of the form *ax*² + *bx* + *c*. The values of *x* for which a quadratic is equal to zero are called the roots of this quadratic. The number of real roots of a quadratic depends on the sign of $\Delta = b^2 - 4ac$ which is called the discriminant.

- If $\Delta > 0$ then the quadratic has two real roots.
- If $\Delta = 0$ then the quadratic has one real root (two equal real roots).
- If $\Delta < 0$ then the quadratic has no real root.

The extremum of a quadratic occurs when x is equal to $-\frac{b}{2a}$.

The **break-even point** is the point at which cost and revenue are equal.

A company manufactures and sells x radios per month.

The cost, C, in dollars, involved in producing x radios per month is given by the equation

$$C(x) = 60x + 70000 \qquad 0 \leq x \leq 6000.$$

The revenue, R, in dollars, based on the sales of x radios per month is given by the equation

$$R(x) = -\frac{1}{30}x^2 + 200x, \qquad 0 \leq x \leq 6000.$$

1. Draw accurately the graphs of the cost and revenue functions on the same set of axes.

2. Calculate:
   a. the minimum cost involved;
   b. the maximum revenue.

3. Why is there a cost involved when no radios are produced?

4. On your graph, identify the break-even points.

5. What profit does the company make when 2000 radios are produced and sold?

6. a) Find an expression in terms of x for the profit, , P, in dollars, this company makes on the sales of their radios.
   b) How many radios would they need to sell to earn $ 60,000?
   c) How many radios would they need to sell to achieve this maximum profit?
   d) What is the maximum profit the company can hope to make?

7. For what values of x will the company be in the red?

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MAPPING
D1 - 73

The first part of this page is a summary that can help you do the exercise.

- Quadratic functions are polynomials in which the largest exponent is 2. The graph of a quadratic function is a parabola, and its equation is $y = ax^2 + bx + c$.

  If $a > 0$, the parabola opens up and has a minimum value.
  If $a < 0$, the parabola opens down and has a maximum value.
  The x-coordinate of the vertex of the parabola is equal to $\frac{-b}{2a}$, and the axis of symmetry is the vertical line whose equation is $x = \frac{-b}{2a}$.

- Given the radius R of a circle, the circumference of a circle can be calculated using the formula : $2\pi R$ and the area using the formula : πR^2

A Norman Window is a window that has the shape of a semicircle on top of a rectangle (the diameter of the circle is equal to the width of the rectangle):
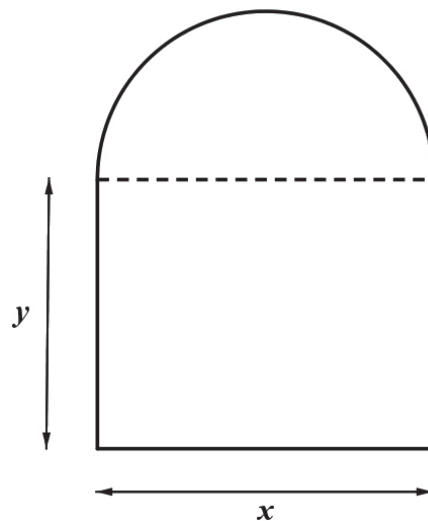
The goal of this problem is to maximize the area of a Norman window with a given perimeter so that it lets in as much light as possible.

1) The **perimeter** of the window is **20 feet.** Write y in terms of x (use 3.14 for π).

2) Show that the area A of the window is equal to :
$$A(x) = -0.8925x^2 + 10x$$

3) Work out the value of x so that the area $A(x)$ is maximum and find the maximum area (rounded to 1d.p.).

MAPPING
D1 - 74

The first part of this page is a summary that can be helpful to do the exercise.

• The standard form equation of a quadratic function is: $f(x) = ax^2 + bx + c$ with a, b and c constants, $a \neq 0$. The graph of f is called a parabola.

The x-coordinate of the vertex of the parabola is $-\dfrac{b}{2a}$.

• The domain of a function f is the set of all the numbers that have an image under f.
The range of a function f is the set of all the numbers that can be written as images of a number under f.

•To solve the quadratic equation : $ax^2 + bx + c = 0$ with $a \neq 0$, compute the discriminant $\Delta = b^2 - 4ac$

->If $\Delta > 0$, the solutions of the quadratic equation are given by the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

If $\Delta = 0$, there is one real number solution $x = \dfrac{-b}{2a}$ called a double root of the equation.

If $\Delta < 0$, there is no real number solution.

Exercise

1. For x in [0;6], let f be the function by: $f(x) = 2x - 1$.
    (*i*) Describe the graph of function f.
    (*ii*) What is the range of function f ?

2. Let g be the function defined for all x by: $g(x) = x^2 - 4x + 3$.
    (*i*) Solve for x the equation $g(x) = -1$, then the equation $g(x) = 3$.
    (*ii*) Describe the graph of function g.
    (*iii*) What is the range of function g ?

3. Do the curves of equation $y = f(x)$ and $y = g(x)$ intersect? How would you find the coordinates of the intersection points?

4. h is a quadratic function, whose range is [5; +∞[, and whose graph passes through A(1; 7).

    (*i*) In your opinion, how many of such functions are there?
    (*ii*) Could you find an expression for $h(x)$ in terms of x for at least one such function?

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MAPPING
Sujet n° D1_81

The first part of this page is a summary that can help you do the exercise.

A function takes one element of a first set ("domain") and assigns it to one, and only one, element of a second set ("range").
The function maps each element of the domain onto its image in the range.
If a function can map more than one element of the domain onto the same element of the range, it is said to be many-to-one. If each element of the range is mapped onto a single element of the domain, the function is said to be one-to-one.

1) The following function maps an element x onto its image $f(x) = y$.
   $f : x \longrightarrow 2x - 3$.
   (a) Find the range of the function for its domain $\{0 \leq x \leq 4\}$.
   (b) Explain why f is a one-to-one function.

2) (a) What is the name of the graph of a quadratic function?
      What shape is that graph?
      Does such a graph have a line of symmetry?

   (b) Function $x \longrightarrow x^2$ has the domain $\mathbb{R}$ because it is defined for all real x.
      (i) Explain why the range will contain no negative number.
      Therefore, the range is $\{y \in \mathbb{R} \,;\, y \geq 0\}$
      (ii) Give an example to show that some elements of the range can be obtained from more than one element of the domain.
      (iii) Is the function many-to-one or one-to-one?

3) (a) Construct a table of values for the function $f : x \longrightarrow 2x - 3$. How many values do you need to plot the graph of f? Explain how you would draw it.
   (b) State whether the point (12.5, 21) lies on the graph.

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MAPPING

Sujet n° D1_82

The first part of this page is a summary that can help you do the exercise.

- Linear function:

A linear function is in the form $f(x) = mx + p$.

If two points $A(x_A, y_A)$ and $B(x_B, y_B)$ − with $x_A \neq x_B$ − belong to the graph of a linear function, the gradient m is given by the formula $\frac{y_A - y_B}{x_A - x_B}$.

- Quadratic function:

A quadratic function is in the form $f(x) = ax^2 + bx + c \; with \; a \neq 0$.

The x-coordinate of the vertex of its graph is equal to $-\frac{b}{2a}$.

To solve $ax^2 + bx + c = 0 \; with \; a \neq 0$, you can use the quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Two brothers live in the same house and their bedrooms are on two different floors. The younger brother's room is above the other one.

At the same time, they throw something in their garden from the window of their own room: the younger throws a stone and the elder a tennis ball.

We want to study the paths of their projectiles in terms of the time.

Let's label t the time, $S(t)$ the path followed by the stone and $B(t)$ the path followed by the tennis ball.

1) Path of the stone:

At $t = 0$ s, the younger child throws a stone which follows a straight line. At $t = 1$ s, the height of the stone is 4 m and at $t = 3$ s, the stone is 2m above the ground.

    a) Find the expression of $S(t)$ in terms of t.

    b) What is the height of the stone at the beginning of the throw?

    c) When does the stone reach the ground?

2) Path of the tennis ball:

The tennis ball follows a quadratic curve.

At the beginning of the throw, the ball is 1 m high. 2s after the beginning, the ball is 5 m high and at $t = 3s$, the ball is 4 m above the ground.

    a) Prove that the path of the tennis ball is given by the expression

        $B(t) = -t^2 + 4t + 1$.

    b) Describe the path followed by the tennis ball. When does the ball reach its maximum height?

    c) When does the ball reach the ground?

3) When are the two projectiles at the same height?

MAPPING
Sujet n° D1_83

The first part of this page is a summary that can help you do the exercise.

A quadratic function is in the form $f(x) = ax^2 + bx + c \text{ with } a \neq 0$.
The x-coordinate of the vertex of its graph is equal to $-\frac{b}{2a}$.

To solve $ax^2 + bx + c = 0 \text{ with } a \neq 0$, you can use the quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Alfred's rectangular garden is composed of a lawn and a path. His garden is 30 by 16 yards. He wants to make a path all around the lawn such that the area of the path is equal to half the area of the garden. The width of the path must be the same everywhere.
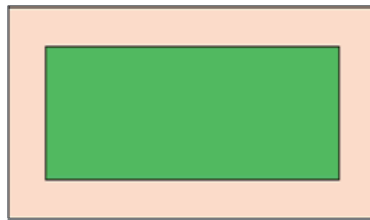
1) What is the area of the garden?

2) Let x be the width of the path. Show that the area of the path is $-4x^2 + 92x$.

3) Show that an equation to find the width of the path is $x^2 - 23x + 60 = 0$.

4) Solve the equation and conclude the problem.

5) Later, he changed his mind and he wants to make two perpendicular paths of the same width like below:
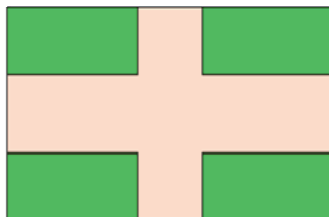
We still want the area of the path to be half the area of the garden. What must the width of the path be?

MAPPING
Sujet n° D1_84

The first part of this page is a summary that can help you do the exercise.

The standard form equation of a quadratic function is: $f(x) = ax^2 + bx + c$ with a, b and c constants, $a \neq 0$. The graph of f is called a parabola ; the abscissa of its vertex is $-\dfrac{b}{2a}$.

Two companies A and B produce and sell 500 g corn flakes packs for £5 each.

1) (a) The cost of production in pounds, for company A, is given for q units produced and sold by:
$$C(q) = 0.01q^2 - 10q + 2510.$$
How many units should be produced and sold so that the cost is minimum?

(b) Let $B(q)$ be the profit in pounds for q units produced and sold.
Explain why $B(q) = -0.01q^2 + 15q - 2510$.

(c) Do you think that the profit is maximum when the cost of production is minimum?

2) The two companies decide to lower their prices: company A offers 10% off the price for each pack, while company B offers 10% more product for the same price of £5. Show that company A gives the better offer.

3) On each pack of corn flakes, company A uses a friendly logo made out of two intersecting parabolas, as shown below. Find the expression of $f(x)$ in terms of x?
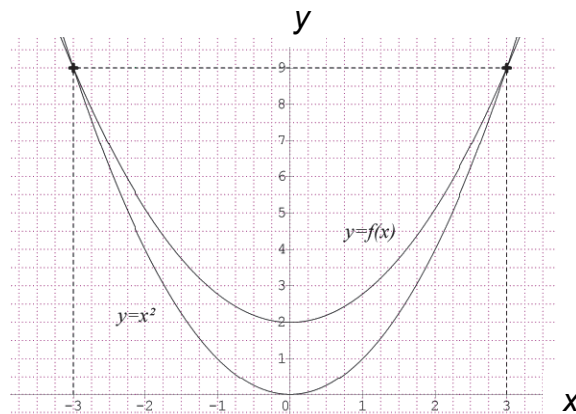
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MAPPING
Sujet n° D1_91

The first part of this page is a summary that can be helpful to do the exercise.

The expanded form of a quadratic function is: $y = ax^2 + bx + c$ with a, b, c constants and $a \neq 0$.
The x-coordinate of the vertex is $\frac{-b}{2a}$.
The factorised form of a quadratic equation with x-intercepts x_1 and x_2 is:
$y = a(x - x_1)(x - x_2)$ with a constant and $a \neq 0$

Exercise :

In the eighties, a famous brand of ice cream decided to modernize the shape of their products (same thickness ($1.5 \, cm$), but a more rounded look). The two mathematical models are given on the right.

1. An equation of the quadratic function drawn on model A is:

$$y = -\frac{2}{9}x^2 + \frac{2}{3}x + 8$$

Describe the shape of the curve, explaining your answer and give the coordinates of its vertex.

2. P_1 is the U-shaped parabola drawn on model B. Read its roots and the coordinates of its vertex graphically. Deduce an equation of P_1.


Before : Model A

After : Model B

3. The area under the adjacent parabola is given by $\frac{2}{3}bh$, where b is the length of the base, and h the height.

   a. One unit is one centimeter on both axes.

   Find the dark grey area drawn on model A.

   HINT: You can divide this area into four parts: two triangles, one rectangle and the remaining part above. *Note that in model A, the sides of the model are two line segments CD and EF with C(-0.5, 0); D(0, 8); E(3, 8) and F(3.5, 0)*

   b. Show that the volume of model A is 43.5 cm³.

   c. Using the same method, show that the volume of model B is 44 cm³.
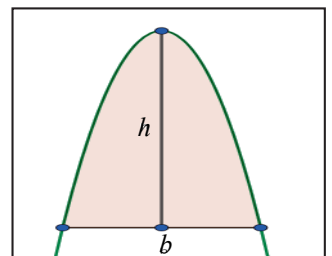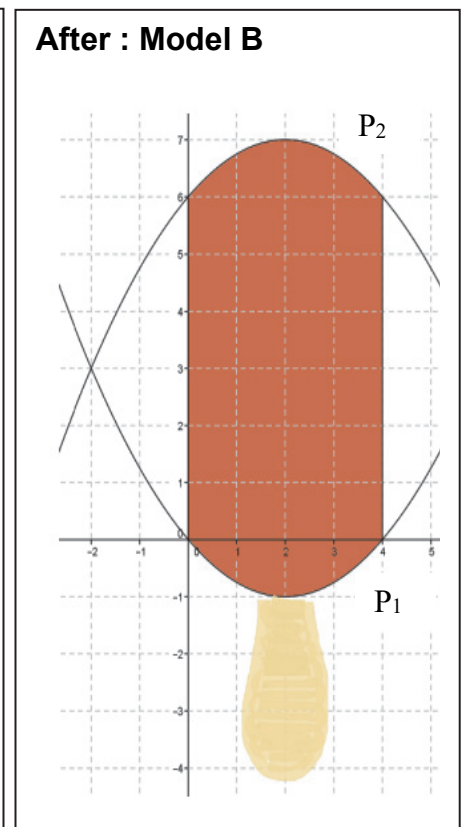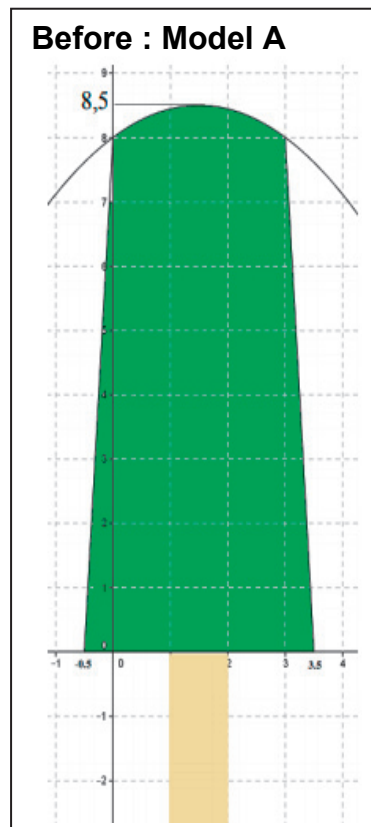
Extra question: The price of the ice-cream has increased by 2%. Do you think that the consumers have been cheated with the change of model?

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DIFFERENTIATION
D2 - 71

Tools
Given a right triangle ABC in B
- The Pythagoras theorem states $AB^2 + BC^2 = AC^2$
- The surface of the triangle is given by $\frac{AB \times BC}{2}$

Product Rule
Given two derivable functions $u(x)$ and $v(x)$ we have $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

An artist has been commissioned to make a stained glass window in the shape of an octagon. He wants to minimize the surface of the glass he will use in order to minimize the costs.
The octagon must fit inside an 4-in. square space. See the figure below :
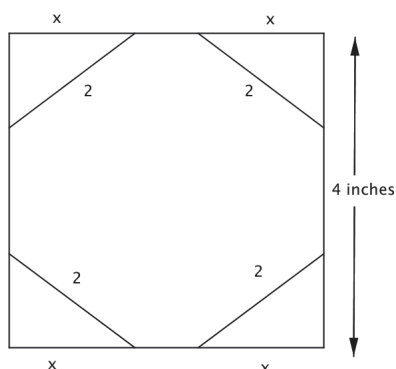
1. The hypotenuse of each triangle must be 2 in. long. Let x be the length of one leg. Explain how to find and compute the expression of the length of the other leg.
2.
   a. Show the area of one triangle must be $A(x) = \frac{x\sqrt{4-x^2}}{2}$.
   b. What is the domain of $A(x)$?
   c. Compute the derivative $\frac{dA}{dx}$.
3. What is the minimum surface area of glass?

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DIFFERENTIATION
D2 - 72

Tools
Surface of a circle, with a radius r : πr^2
Perimeter of a circle with a radius r : $2\pi r$

Liam wants to order cylindrical cups for his birthday party from a cup factory.
<u>The external and base surface</u> area must be 48π cm^2.
This diagram shows a cylindrical container of radius r cm and height h cm. The container has an open top and a circular base.
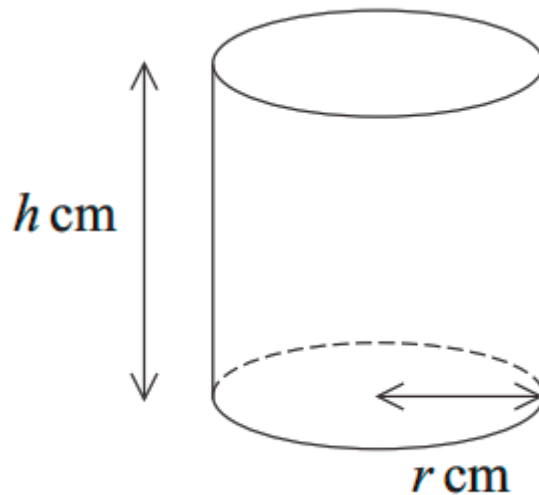
When the radius of the base is r cm, the volume of the container is V cm^3.
Liam wants to find if there are values for h and r that maximize the volume of the cup and what they are if they exist.

1) Show that $h = \frac{48-r^2}{2r}$ and deduce that $V = 24\pi r - \frac{\pi}{2}r^3$.
2)
    a. Find the derivative $\frac{dV}{dr}$.
    b. Find the values of r for which V has a stationary point.
3) In order to maximize the volume find the dimension of the cups that Liam must ask for.

After AQA-General Certificate of Education Advanced Subsidiary Examination, June 2015.

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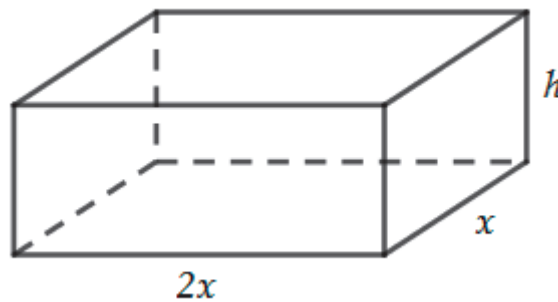Académies de Paris-Créteil-Versailles

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Differentiation

D2 - 81

Tools

- The volume of a rectangular parallelepiped is $V = a \times b \times c$ where a is the width, b the length and c the height (all expressed in the same unit).

- The total surface area is equal to the sum of the surface areas of all the faces.

The diagram shows a block with a base measuring $2x$ cm by x cm and a height of h cm.

The total surface area of the block is 300 cm^2.

1) Show that the total surface area of this block is: $S = 4x^2 + 6hx$.

2) Show that $h = \dfrac{150 - 2x^2}{3x}$.

3) Express the volume of the block in terms of x.

4) Find the value of x that gives the block a maximum volume and calculate this maximum volume.
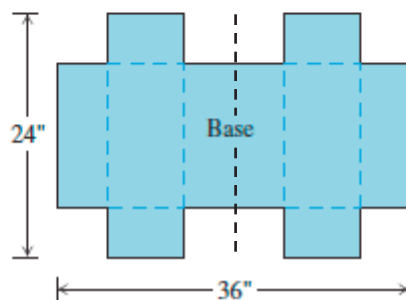
DIFFERENTIATION
D2 - 82

Designing a Suitcase

A 24- by 36-in. sheet of cardboard is folded in half to form a 24- by 18-in. rectangle as shown in the figure.
Then four congruent squares of side length x are cut from the corners of the folded rectangle. The sheet is unfolded, and the six tabs are folded up to form a box with sides and a lid.
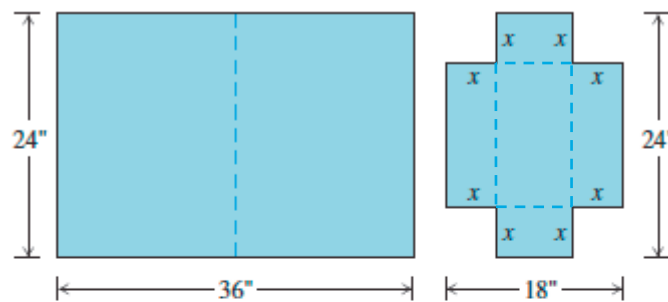
The sheet is then unfolded.

1. Write down a formula for the volume $V(x)$ of the box.
2. Check that $V(x) = 8x(x^2 - 21x + 108)$.
3. Find the domain of V for the problem situation.
4. Using your calculator, plot V over its domain. (hint : $X_{min} = 0$, $X_{max} = 10$, $Y_{min} = 0$ and $Y_{max} = 1400$)
5. Use the graph to find an approximate value of the maximum volume and the value of x that gives it.
6. Confirm the result you found in the previous question by a calculation.
   (hint: differentiate V)
7. Find a value of x that yields a volume of $1100 \ in^3$.

DIFFERENTIATION

D2 - 91

Tools

- The distance, in metres, run by an object after *t* seconds is given by $f(t)$ then its speed after t seconds is given by $\frac{df}{dt}$ and its acceleration by $\frac{d^2f}{dt^2}$.
- The equation of the tangent line at t_0 is given by $y = \frac{df}{dt}(t_0)(t - t_0) + f(t_0)$

An arrow is shot upwards from the window of a castle.

After t seconds, its height above the ground is given by

$$h(t) = -3t^2 + 30t + 15.75 \qquad \text{in metres.}$$

1) How high is the window?

2) Show that the arrow reaches the ground after 10.5 s.

3) Work out the tangent line of its trajectory

    a. After 2 s

    b. After 7 s.

4) When is the speed equal to zero?

   Give an interpretation.

5) Find the acceleration of the arrow.
   What is your interpretation of its sign?

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SEQUENCES
Sujet D3 - 52

The first part of this page is a summary that can help you do the exercise.

In a geometric sequence, each term can be obtained by multiplying the previous term by a constant value. This value is called the common ratio, or r.

You can find the nth term of a geometric sequence using the formula $u_n = u_1 \times r^{n-1}$.

On January the 1st, Harry invests £2000 in an account which pays 5% interest per month.

At the start of each month, starting from February, he spends £200. The interests are paid on the 20th of each month.

Let u_n be the amount in his account **at the end** of the nth month (January is the first month).

1. Find the values of u_1 and prove that $u_2 = 1995$.

2. Express u_{n+1} in terms of u_n.

3. (V_n) is the sequence defined by $V_n = u_n - 4200$.

   a. Prove that (V_n) is a geometric sequence and find its common ratio and the value of V_1.

   b. Express the nth term V_n in terms of n and then the nth term of u_n in terms of n.

4. After how many months will he be in debt?

5. Does such an interest rate seem realistic?

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SEQUENCES

Sujet D3 - 62

The first part of this page is a summary that can be useful to do the exercise.

Sequences

A geometric progression (G.P.) is a sequence in which each term after the first is formed by multiplying the previous term by a fixed number, called the common ratio.
If a_1 is the first term and r is the common ratio, the nth term is: $a_n = a_1 \times r^{n-1}$.

An arithmetic progression (A.P.) is a sequence in which each term after the first is formed by adding a fixed number to the previous term, called the common difference.
If a_1 is the first term and r is the common difference, the nth term is: $a_n = a_1 + (n-1)r$.

Exercise:

Thomas Malthus (1766 – 1834) was an English cleric and scholar, influential in the fields of political economy and demography. In his Essay on the Principle of Population (first edited in 1798), based on statistical figures of the 18th century, he argued that population multiplies geometrically and food arithmetically; therefore, the population will eventually outstrip the food supply.

In 1801, the population in England was of 8 million inhabitants and the rate of growth of the population was 2% per year. Meanwhile, the British agriculture was able to feed 10 million inhabitants, and Malthus estimates each year, it could feed an extra 0.4 million inhabitants.

For $n \geq 1$, let P_n be the population in year $1800 + n$, and let F_n be the population that can be fed in year $1800 + n$.

1. Explain why P_n is a geometric sequence. Give the common ratio, the first term and expression of the nth term.
2. Explain why F_n is an arithmetic sequence. Give the common difference, the first term and expression of the nth term.
3. According to Malthus, what will be the population in England in 1850? How many inhabitants can be fed in England in 1850?
4. Use any method of your choice to find out when it's not possible to feed the whole population anymore.

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SEQUENCES

Sujet D3 – 63

Summary :

Arithmetic sequence :

A sequence $\{a_n\}$ is an arithmetic sequence (or arithmetic progression) if it can be written in the

form : $a_n = a_{n-1} + d$, where $n \geq 2$, for some constant *d*.

The number *d* is the common difference.

Let a_1 be the first term.

The *n*th term of an arithmetic sequence can be written in the form $a_n = a_1 + (n-1)d$

Arithmetic series :
A series is an arithmetic series if its terms form an arithmetic sequence.

The sum S_n of the arithmetic series $a_1 + a_2 + a_3 + \ldots + a_k + \ldots + a_n$ with common difference *d* is

$$S_n = n\left(\frac{a_1 + a_n}{2}\right) \text{ or } S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

Exercise:
We draw a succession of interlocked pentagons P₁, P₂, P₃, P₄ as shown below:

u_n is the number of dots in each figure, it is called the nth pentagonal number.
$u_1 = 1$
$u_2 = 5$
$u_3 = 12$

1. Give the value of u_4.
2.
a. How many dots are there on one side of the nth pentagon P$_n$?
b. Prove that $u_{n+1} = u_n + 3n + 1$. If you cannot prove it, move on to the next question.

3. For n, $n \geq 1$, $v_n = u_{n+1} - u_n$.
a. Prove that (v_n) is an arithmetic sequence (A.P).
b. Give $v_1 + v_2 + v_3 + \ldots + v_{n-1}$ in terms of n.
c. Write $v_1 + v_2 + v_3 + \ldots + v_{n-1}$ in terms of u_n . Then, give the expression of u_n in terms of n.

4. Give the 10th pentagonal number.

SEQUENCES

Sujet D3 - 65

The first part of this page is a summary that can help you do the exercise.

Arithmetic sequence

A sequence (a_n) is an arithmetic sequence with common difference d if it can be written $a_{n+1} = a_n + d$, where $n \geq 1$.

The n-th term of an arithmetic sequence whose first term is a_1 and common difference d can be written $a_n = a_1 + (n-1)d$.

The sum of the first *n* terms of an arithmetic sequence is: $S_n = \dfrac{n}{2}(a_1 + a_n)$.

Geometric sequence

A sequence (a_n) is a geometric sequence with common ratio r if it can be written $a_{n+1} = r.a_n$, where $n \geq 1$.

The n-th term of a geometric sequence whose first term is a_1 and common ratio r can be written $a_n = a_1\, r^{n-1}$.

When r ≠ 1 the sum of the first n terms of a geometric sequence is $S_n = a_1 \times \dfrac{1-r^n}{1-r}$.

A **well** is a deep hole made in the ground through which water can be removed.

EXERCISE

A company wants to drill a deep well into the ground, and contacts two businesses A and B.

With business A, the first metre to be drilled costs £50, and each extra metre costs £10 more than the previous one.

With business B, the first metre to be drilled costs £40, and each extra metre costs 10% more than the previous one.

Business A

1. Let a_n be the price of the nth metre to be drilled into the ground by business A.

  (a) Find a_1, a_2, and a_3; what can you say about the sequence (a_n)?

  (b) Write a_n in terms of *n*.

2. Let A_n be the price of a drilling of total length n metres with business A.

  (a) Find A_1, A_2, and A_3; what can you say about the sequence A_n?

  (b) Explain why $A_n = n(5n+45)$.

Business B

3. Let b_n be the price of the *n*th metre to be drilled by business B, and B_n the price of a *n*-metre drilling with business B. Find b_n in terms of *n*. Show that $B_n = 400(1.1^n - 1)$

Business A or business B?

4. The company has to drill a 43-metre-long well into the ground: could you figure out which offer seems to be the best and what the final cost would be?

SEQUENCES
D3 - 71

A sequence in which the differences between successive terms are equal is called an arithmetic sequence or arithmetic progression.
The number added to each term to get the next one is called the common difference.
If the initial term of an arithmetic progression is a_1 and the common difference is d, then the nth term of the sequence, called the general term, is given by: $a_n = a_1 + (n-1)d$.
$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

Jeremy, a bee-keeper, is doing an investigation which involves building hexagon-shaped patterns with sticks. His first three patterns are shown below.

Step 1   Step 2   Step 3
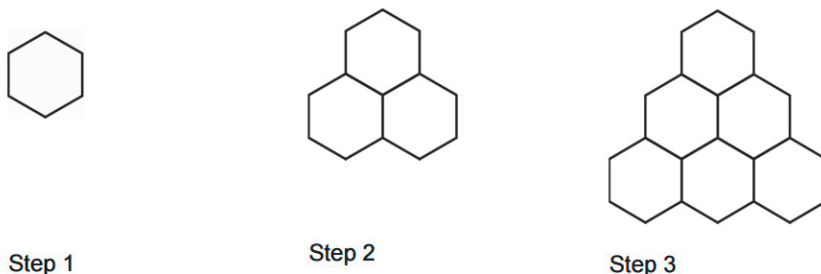
1) What is the extra number of hexagons and the extra number of sticks necessary in step 4?

2) Copy and complete the table below:

Step n	1	2	3	4
Number of hexagons on row n: h_n	1	2		
Total number of hexagons at step n: H_n	1			
Extra number of sticks on row n: T_n	6	9		
Total number of sticks used at step n: S_n	6			

1) **Number of hexagons**: express the total number of hexagons at step n in terms of n. Explain your answer.

Number of sticks:
2)
   a) Using the table above, express T_{n+1} in terms of T_n. What type of sequence is T_n?
   b) Express T_n in terms of n.

3) Express S_{n+1} in terms of S_n and n.

4) Jeremy would like to find a formula to express S_n in terms of n. Here are three formulae.
   Help him choose the right one. (Explain your choice)
   $$S_n = 2n^2 + 4n \qquad S_n = \frac{3}{2}n^2 + \frac{9}{2}n \qquad S_n = \frac{5}{2}n^2 + \frac{3}{2}n + 2$$

5) Jeremy used 105 sticks to build his last pattern. How many rows are there at this step? How many hexagons are there?

SEQUENCES
D3 - 72

Arithmetic sequences

A sequence (a_n) is an arithmetic sequence if it can be written in the form $a_n = a_{n-1} + d$ where $n \geq 1$. Number d is called the common difference.

The nth term of an arithmetic sequence can be written in the form $a_n = a_1 + (n-1)d$, where a_1 is the first term and d is the common difference.

The sum S_n of an arithmetic series $a_1 + a_2 + a_3 + \cdots + a_k + \cdots + a_n$ with $d \neq 1$ is $S_n = n \times \frac{a_1 + a_n}{2}$.

In your new 'get-fit' program, you plan to jog 1,500 metres on the first night and then increase this distance by 250 metres each subsequent night.

1. First, focus on the distance jogged each night, and complete the table.

Night number	1	2	3	4
Distance jogged (m)				

2. If you continue the pattern, write down an expression for D_n, the distance jogged on the nth night.

3. How far will you jog :
a) on the 7th night ?
b) on the 12th night ?

4. Now focus on the total distance jogged over several nights, and complete the table.

Night number	1	2	3	4
Total distance jogged (m)				

5. Write down an expression for S_n, the total distance jogged after n nights.

6. Determine the total distance you expect to jog after ten nights.

7. Determine the number of nights you will need to ensure that you jog a total of more than 50km.

SEQUENCES

D3 - 81

Arithmetic and geometric sequences

The nth term of an arithmetic sequence can be written in the form $a_n = a_1 + (n-1)d$, where a_1 is the first term and d is the common difference.

The nth term of an geometric sequence can be written in the form $b_n = b_1 * q^{n-1}$, where b_1 is the first term and q is the common difference.

The sum S_n of the arithmetic series $a_1 + a_2 + a_3 + \cdots + a_k + \cdots + a_n$ with $d \neq 1$ is $S_n = n \times \frac{a_1 + a_n}{2}$.

The sum of the geometric series $b_1 + b_2 + \ldots + b_n = b_1 \times \dfrac{1-q^n}{1-q}$

The third labour of Heracles: the capture of the Ceryneian Hind

The Greek hero Heracles (or Hercules) had to carry out 12 extraordinary labours given by King Eurystheus in order to expiate one terrible crime.
The third labour was to capture the Ceryneian Hind, a hind so fast it could outrun an arrow. As Heracles finds the hind, the startled animal starts to run and Heracles has no choice but to chase it.

- The first day, the Ceryneian Hind runs 50 kilometers and since it is chased, each day it will run 10 more kilometers than the day before.
- The first day, Heracles will run 40 kilometers and since he really needs to catch the hind, each day he will run a 5% longer distance than the day before.

Let u_n denote the distance run by the hind on the n-th day and v_n denote the distance run by Heracles on the n-th day. So $u_1=50$ and $v_1=40$

1. Calculate the distance run by the Ceryneian Hind and Heracles the second day.

2. Calculate the distance to the nearest kilometer run by the Ceryneian Hind and Heracles the tenth day.

3. Find out smallest index n for which $u_n \leqslant v_n$. What does that mean?

4. Using your calculator, find out how many days Heracles needs to catch up with the Ceryneian Hind. Calculate the total distance to the nearest kilometer run by the animal and the hero before he manages to capture it.

SEQUENCES
D3 - 82

The first part of this page is a summary that can be useful to do the exercise.

Sequences

A geometric progression (G.P.) is a sequence in which each term is obtained by multiplying the preceding term by a fixed number, called the common ratio.
If a_1 is the first term and r is the common ratio, the nth term is: $a_n = a_1 r^{n-1}$.

An arithmetic progression (A.P.) is a sequence in which each term is obtained by adding a fixed number, called the common difference, to the preceding term.
If a_1 is the first term and r is the common difference, the nth term is: $a_n = a_1 + (n-1)\, r$.

Exercise

On a tweeting social network, Albert realizes that every morning, he gets 10% more followers than the day before, but then 5 followers change their minds and unfollow him during the rest of the day.
Let (a_n) be the number of followers Albert has at the end of the nth day.

1. Explain why $a_n = 1.1 a_{n-1} - 5$, for every integer n.

2. If Albert has 50 followers at the end of the first day, how many friends will he have by the end of the month (30 days later)? By the end of the year (365 days later)?

3. What happens if Albert has 50 friends at the end of the first day?

4. What happens if Albert has less than 50 friends at the end of the first day?

5. We assume that Albert has 51 friends at the end of the first day.

   a. Let (u_n) be a sequence such that $u_n = a_n - 50$ for every integer n.

      Explain why (u_n) is a geometric progression.

   b. Find the expression (only in terms of n and u_1) of the general term of (u_n).

      Prove that $a_n = 1.1^n + 50$.

   c. There are 300 million active members on the tweeting social network. When will Albert be theoretically followed by all of the active members?

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SEQUENCES

D3 - 83

The first part of this page is a summary that can be useful to do the exercise.

Sequences

A geometric progression (G.P.) is a sequence in which each term is obtained by multiplying the preceding term by a fixed number, called the common ratio.
If a_1 is the first term and r is the common ratio, the nth term is: $a_n = a_1 r^{n-1}$.

An arithmetic progression (A.P.) is a sequence in which each term is obtained by adding a fixed number, called the common difference, to the preceding term.
If a_1 is the first term and r is the common difference, the nth term is: $a_n = a_1 + (n-1)\, r$.

Exercise

One fine day, Mary and John chose two different ways of saving money.

- John first put £500 into a box and decided to add £150 each month.
- Mary first put £5 into her own box, deciding to double its content every month.

Let us call j_n and m_n the total amount of money John and Mary have respectively saved after n months.

1. What are the values of j_1 and m_1?
2. How much money will each of them have at the beginning of the 2nd month? At the beginning of the 3rd month?
3. Express j_n in term of n and j_1.
4. Express m_n in term of n and m_1.
5. Prove that John will have saved £2,300 and Mary £20,480 after one year,
6. Use your calculator to find the first month when Mary will become richer than John.

Domain 3 : SEQUENCES

D3 - 91

> **Reminder:** The general term of an arithmetic sequence with first term a_1 and common difference d is given by $a_n = a_1 + d(n-1)$.

Exercise: Here is a sequence of diagrams made using identical rectangles. A dot is shown at the junction of three lines. A cross is shown at the junction of two lines.

Diagram 1   Diagram 2   Diagram 3   Diagram 4
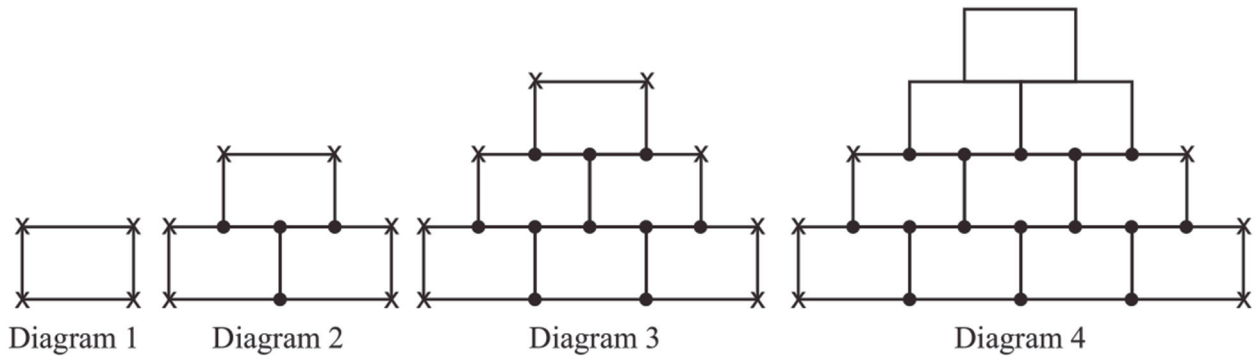
1/ a/ How many dots and crosses are missing in diagram 4?

1/ b/ Complete the table for Diagram 4 and Diagram 5.

Diagram	1	2	3	4	5
Number of dots	0	4	10		
Number of crosses	4	6	8		

2/ a/ Describe the evolution of the number of dots from one diagram to the next.

2/ b/ Let D_n be the number of dots in Diagram n.

Express D_{n+1} in terms of D_n and n.

2/ c/ For $n \geq 1$, $D_n = n^2 + n - 2$.

Find the number of dots in Diagram 12.

3/ a/ Write down an expression for the number of crosses in Diagram n.

3/ b/ Diagram n has 100 crosses. Find the value of n.

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STATISTICS
Sujet D4 - 51

Stem & leaf and Histogram

The first part of this page is a summary that can help you do the exercise.

Reminder :
Let $(x_1, x_2, \ldots, x_N)$ be a distribution of N numbers.
The **mean** is given by $\frac{x_1 + x_2 + \cdots + x_N}{N}$.

If N is even, then the **median** is the mean of the two middle values.
If N is odd, then the **median** is the middle value.
The **first quartile** Q_1 is given by x_I where I is $\frac{N}{4}$, rounded to the next integer.

The **third quartile** Q_3 is given by x_J where J is $\frac{3N}{4}$, rounded to the next integer.
The **modal value** is the most common value.
A **histogram** displays frequencies from a grouped frequency distribution: the frequency of a class interval is represented by a rectangle whose surface area is proportional to the frequency.

A group of 20 students took a math test. The school board had the tests graded by two applicant teachers (Ap1 and Ap2). The grades are shown in the back-to-back stem-and-leaf diagram below.
The board of directors would like to hire one math teacher.
Here are the marks :

Ap1			Ap2
1	1		
	2		
	3	9	
	4	4 5 7	
9 8 7 3 0	5	3 4 4	
9 8 7 7 6 3	6	0 1 3 6 7 9	
5 4 3 2 1 1 1	7	2 2 3 5	
0	8	0 5	
	9	3	

key: 7 | 4 | means 47   key: | 8 | 0   means 80

inspired from "Concise Maths 4&5 – for Leaving Cert higher level – Project maths supplement"
by George Humphrey – Brendan Guildea – Geoffrey Reeves – Louise Boylan
Gill & MacMillan editor 2010

Questions

1. Did the first applicant give the mark 68? And did Ap2 give the mark 46?

2. Compute the mean, median, interquartile range for both distributions.

3. Find the modal values. Explain how to retrieve them from the stem-and-leaf plot.

4. From the measurements that you have computed, would you rather hire Ap1 or Ap2? Explain the reasons for your choice.

5. Draw a histogram for the grades given by Ap2. Describe the relationship between the stem-and-leaf plot and the histogram.

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STATISTICS
Sujet D4 - 53

The first part of this page is a summary that can help you do the exercise.

Let's consider a set of numerical values.
▸ The **mode** is the most common value.
▸ The **mean** is the sum of all values divided by the total number of values.
▸ The **median** is the middle value when the values are listed in order. If there are two middle values (because there is an even number of values) the median is the mean of these two middle values.
▸ The **range** is the difference between the highest value and the lowest value.

EXERCISE   *Questions 1 and 2 are independent, and can be answered separately.*

Question 1   The best route

James has two possible ways of getting to school:
A: He can catch a bus to the town centre and another bus out; *or*
B: He can walk in a different direction and then just catch another bus.

He isn't sure which way is quicker so he decides to time his journey from leaving his front door to arriving at the school gate. He times 20 journeys by route **A** and 20 by route **B**.
Here are his results (in minutes).

Route **A**	24	18	14	19	22	23	18	15	22	30
	30	26	22	27	16	22	26	19	19	27

Route **B**	17	17	18	18	18	19	20	20	21	21
	21	22	24	24	24	24	25	25	26	28

a) The mean, median, mode and range for the times of route **A** have already been calculated (see table below). Work out these measures of average and spread for route **B**.

route	mean	median	mode	range
A	21.95	22	22	16

b) Which route do you think James should use, and why?

Question 2   Possible or Impossible?

I have a set of numerical values which could be anything, and I want to know whether each of these statements (separately) is possible or impossible.
Can it apply to the mean, the median, the mode or the range?
If it's possible, give an example; if impossible, explain why (some answers are given to help you).

Statement	Mean	Median	Mode	Range
It's equal to zero.		*Possible; e.g. -10, -1, 0, 2, 3*		
It's the highest value.				
It's less than any of the values.			*Impossible, because the mode must be an actual value.*	

Source: Data, numeracy and ICT, Colin Foster (Nelson Thornes, 2003)

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STATISTICS
Sujet D4 – 54

The first part of this page is a summary that can help you to do the exercises.

Recap: 1 inch = 2.54 cm; 1 pound = 0.454 kg

The **mean** of a set of numbers is the sum of the numbers divided by the quantity of numbers.

The **correlation coefficient** measures the strength of the linear relationship between two sets of data.
If the points on a scatter graph lie close to a straight line, a **strong correlation** is said to exist and the correlation coefficient is near 1 (with a positive correlation) or –1 (with a negative correlation).

You may use your calculator to find the correlation coefficient and the least squares regression line in statistics.

Exercise:
The heights and weights of a group of 9 pupils are recorded in the following table:

Height (inches)	64	68	65	55	75	58	72	60	63
Weight (pounds)	134	142	137	124	150	128	147	129	132

1) Calculate the mean height of the group and give your answer in m.

2) Calculate the mean weight of the group and give your answer in kg.

3) Explain how to draw a scatter graph for this data and draw it as best as you can.

   Is it appropriate to fit a straight line for this data?

4) How strong is the correlation between the height and the weight?

5) Draw the line of best fit on your scatter graph.

6) Predict the weight of a pupil who would be:

   (i)   66 inches high   (ii) 39 inches high

7) Why might the previous answer be unreliable?

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STATISTICS
Sujet D4 – 55

The first part of this page is a summary that can help you to do the exercises.

In a list of numbers in order of size, the **median** is the middle number.
If there are *n* numbers in the list, with *n* being an even number, the **median** is half the sum of the $\left(\frac{1}{2}n\right)^{th}$ number and the $\left(\frac{1}{2}n+1\right)^{th}$ number.
The **mean** of a set of numbers is the sum of the numbers divided by the quantity of numbers.

The **correlation coefficient** measures the strength of the linear relationship between two sets of data.
If the points on a scatter graph lie close to a straight line, a **strong correlation** is said to exist and the correlation coefficient is near 1 (with a positive correlation) or –1 (with a negative correlation).

Exercise:

1) The times spent by a class of 30 pupils revising a Math test has been collected in the following table.

Time spent (in minutes)	Less than 10	10-20	20-30	30-40	40-50	50-60
Number of pupils	2	4	8	6	7	3
Cumulative frequency	2					

   a. Fill in the third row of the table above and draw a cumulative frequency curve to estimate the median time, the lower quartile and the upper quartile. Comment your answers.
   b. Find the interquartile range. Comment your answer.

2) We want to know if there is a relationship between the number of minutes spent revising the test and the number of mistakes that are made. A random sample of 9 pupils is listed below.

Time spent (in minutes)	8	18	24	27	33	35	37	41	54
Number of mistakes	12	9	7	7	6	12	4	3	1

   a. Plot the corresponding points in a scatter graph.
   b. What type of correlation is shown?
   c. Draw a line of best fit on your scatter graph.
   d. A pupil revised for 25 minutes. Estimate the number of mistakes he made.

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STATISTICS
Sujet D4 - 61

Part A
A math teacher, Mr. Mat, registered the results of his students in the following table :

Mark x	$0 \le x < 5$	$5 \le x < 10$	$10 \le x < 15$	$15 \le x \le 20$
Frequency	4	8	12	6

1. What type of data is this?
2. Explain how you can find the median of this data.
3. An estimate of the median is 11. Explain what it means.
4. Prove that the interquartile range is roughly equal to 7.

Part B
Mr. Mat wants to compare the results of his students with the marks of his colleague's class (Mrs Smith's) for the same test. The median she found is 13, and the interquartile range she had is 4. Compare the results of the 2 classes.

Part C
Mr. Mat also wants to know if there is a link between the math results of 5 of his students and their English results. This is the table which shows the means of these students in math and English:

Math mean x	12	14	9	11	16
English mean y	11	16	9	12	17

1. This scatter graph illustrates these results:
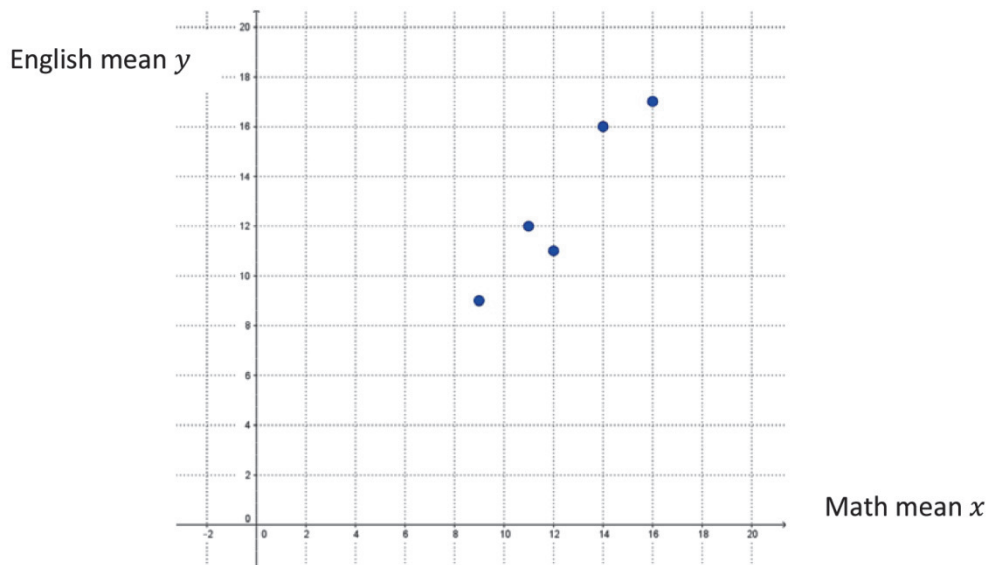
Explain the correlation between the math and the English results.
2. A student has got a math mean of 13. Use the line of best fit to predict his English mean.
3. Another student has got an English mean of 2. Could you predict his math mean?

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STATISTICS
Sujet D4 – 71

The first part of this page is a summary that can help you to do the exercises.

> The frequency density is equal to the frequency of a class interval divided by its width.
> For continuous data, you obtain the median thanks to a cumulative frequency curve: it is the x-coordinate of the point whose y-coordinate is half of the total frequency.
> You can use your calculator to find the correlation coefficient and the least squares regression line in statistics.

Exercise: The following histogram gives the weights of 30 pupils in a class.
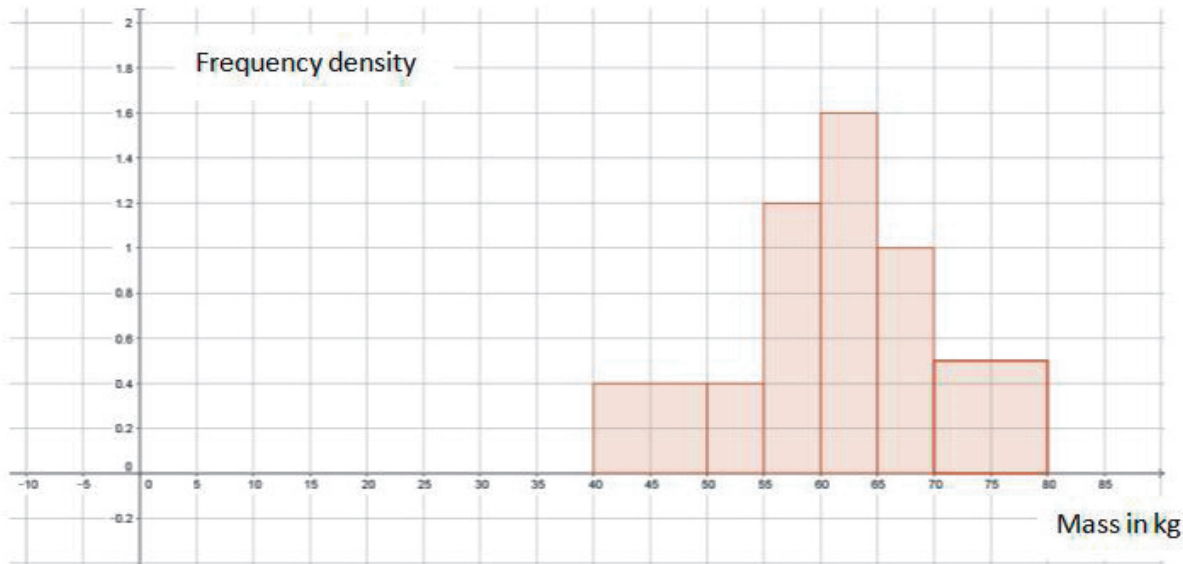
1) What type of data is it?
2) Prove that the frequency of the 1st class interval ($40 \leq m < 50$) is 4.
   Then determine the frequency of each class interval from the graph.
3) Which graph do you have to draw to find the median? Find its value.
4) Interpret the median in the context of the exercise.
5) The heights of the 6 heaviest pupils are given in the following scatter graph.
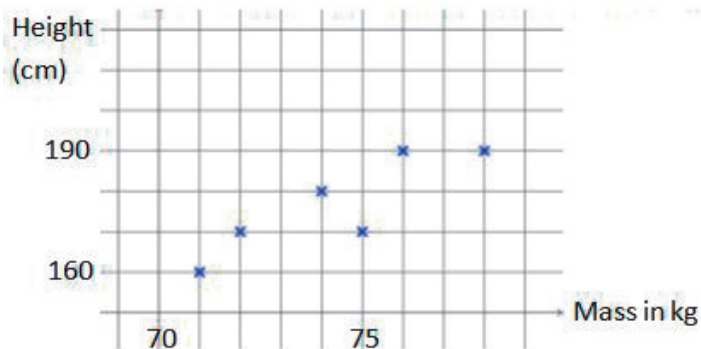
   Is there a correlation between the mass and the height of the pupils?
6) Use the line of best fit to guess the height of a pupil whose weight is 73 kg.
7) Can you predict the weight of a child who is 120 cm tall?

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STATISTICS
Sujet D4- 72

The first part is a summary that can help you do the exercise.

When repeated observations are made on a variable, the result is a frequency distribution. A frequency distribution is recorded in a frequency table. The frequency table gives the possible values or class intervals of the variable and the corresponding frequencies.

A histogram illustrates a frequency distribution. It consists of rectangles drawn on a continuous base. The area of each rectangle is proportional to the frequency of the class it represents. The frequency density is given by the formula: $cy\ density = \frac{frequency\ of\ class\ interval}{width\ of\ class\ interval}$.

You find the cumulative frequency by adding each frequency to the sum of all preceding frequencies. Cumulative frequency provides a convenient way of estimating a median (when the distribution is split into 2 parts), quartiles (when the distribution is split into 4 parts) without considering the raw data.

The histogram below shows the distribution of house prices in an area of Manchester. Prices are given in thousands of pounds (to the nearest thousand).
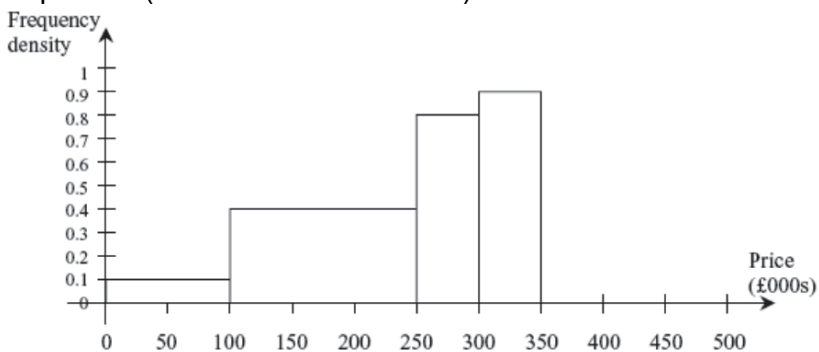
Price in thousands of pounds	0 ≤ x < 100	100 ≤ x < 250	250 ≤ x < 300	300 ≤ x < 350	350 ≤ x < 500
Frequency					60
Cumulative frequency					

1) Explain in your own words how you can complete this histogram (without drawing on the exercise).
2) Copy the table above on your draft paper and complete with the frequencies for each class interval.
3) Work out an estimate of the mean price of the houses. (*Give your result to the nearest pound*)
4) Work out the cumulative frequencies.
5) Draw the cumulative frequency graph on your draft paper using a scale of 1 cm for £50,000 on the x-axis and 1 cm for 20 houses on the y-axis.
6) Using this graph, give an estimation of the median and the interquartile range.
Explain precisely how you find these parameters.
7) How many houses are sold more than £400,000?

Adapted from IGCSE

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STATISTICS
Sujet D4_81

Recap: You can use your calculator to find the correlation coefficient: it allows you to justify the correlation between two variables. The calculator also gives the line of best fit which helps you to make predictions.

For a continuous data, you obtain the median thanks to a cumulative frequency curve: it is the x-coordinate of the point whose y-coordinate is half of the total frequency.

Exercise:
We want to study how the children of a family use their mobiles.
Thanks to a survey, we obtained the following table:

	Child 1	Child 2	Child 3	Child 4	Child 5
age	7	7	8	12	15
Number of SMS/day	10	15	16	30	35

1) We are first interested in finding a link between the age of a child and of the number of SMS sent per day.
   a) Explain how you can draw a scatter graph to illustrate this table, and draw it on your draft paper.
   b) Is there any correlation between the age of a child and the number of SMS he/she sends per day? Justify.
   c) Explain how you draw the line of best fit on your scatter graph.
   d) How old would a child who sends 20 SMS per day be?
   e) Could you predict the number of SMS sent per day for someone who is 50 years old?

2) Calculate the mean number of SMS sent per day in this family.

3) Calculate the median number of SMS sent per day in this family.

4) Let's deal with John, the eldest child. We studied the number of photos taken per day in January. We obtained the following cumulative frequency curve.
   Find the median number of photos taken per day and explain what it means.

Cumulative frequency
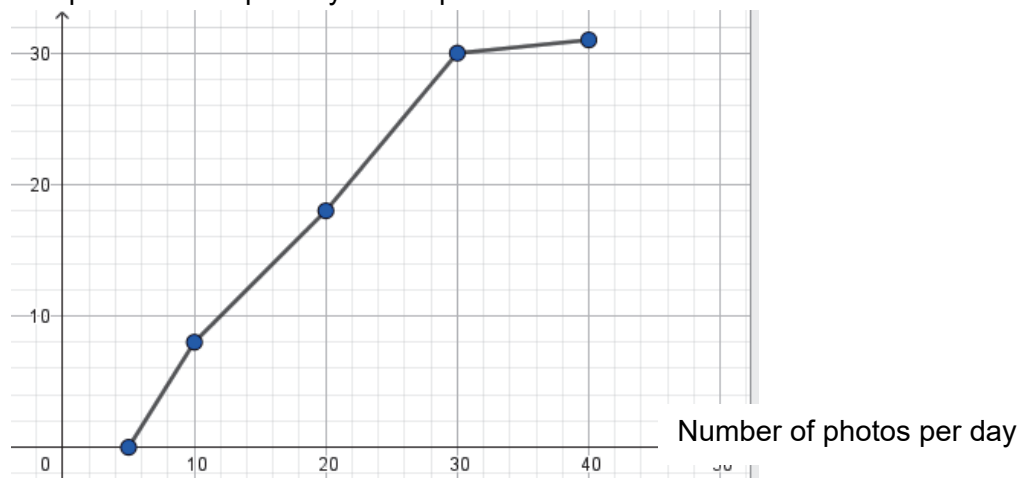
Number of photos per day

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## STATISTICS
Sujet D4-82

Recap

A **linear equation** of a straight line is given by $y = ax + b$, **where a is the slope and b the y-intercept.**

The graph represents the winning time for the men's 100-m freestyle swimming for selected Olympic games.
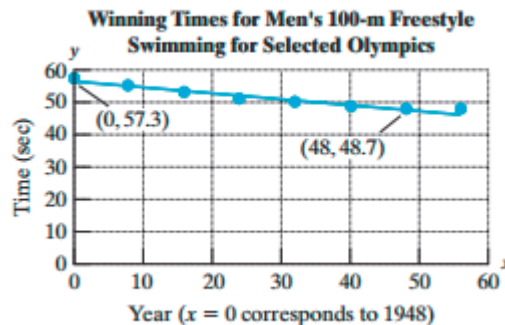
Let y represent the winning time. Let x represent the number of years since 1948 : $x = 0$ corresponds to the year 1948, $x = 4$ corresponds to 1952, and so on.

1. Use the ordered pairs given in the graph $(0; 57.3)$ and $(48; 48.7)$ to find a linear equation to estimate the winning time for the men's 100-m freestyle in terms of the year. Round the slope to 2 decimal places.

2. Use the linear equation from question 1 to approximate the winning 100-m time for 1972, and compare it with the 1996 winning time.

3. Use the linear equation to approximate the winning time for the year 1988.

4. What is the slope of the line and what does it mean in the context of this problem?

5. Explain why the men's swimming times will never reach the $x-$intercept.

6. Do you think this linear trend will continue for the next 50 years, or will the men's swimming times begin to level off at some time in the future? Explain your answer.

STATISTICS

D4-83

Exercise 1

In a survey, two groups of teenagers wrote down how many hours of television they watched in one week.
Here are the box-and-whisker diagrams for the two groups.
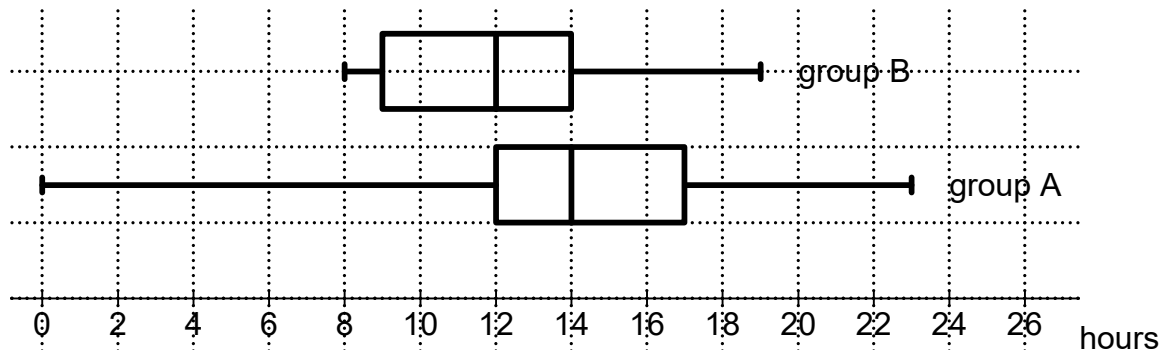
(a) Fill in the blanks in this sentence:
About half of the teenagers in group A watched TV at least … hours that week, whereas about three quarters of the teenagers in group B watched TV no more than … hours.

(b) Write a similar sentence, comparing groups A and B, using the lower quartile of group A.

(c) Work out the interquartile range of hours of TV for each group.

(d) Did the teenagers in group B spend more time watching TV than in group A?

Exercise 2

These are the scores of two players over six rounds of golf:

Ian	87	69	80	86	84	81
William	77	91	90	85	67	71

Remember: the lower the score, the better the player.

(i) Calculate the mean score for each player, to 1 decimal place.

(ii) The standard deviations of the scores are $\sigma_{Ian} \approx 5.9838$ and $\sigma_{William} \approx 9.1727$.
Round these standard deviations to the nearest whole number.
Use the data to state who the better player is. Give a reason for your answer.

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STATISTICS
Sujet D4_84

Exercise 1

Two workers iron clothes. Each worker irons 10 items, and records the time it takes him to iron each item, rounded to the nearest minute:

Worker A: 3   5   2   7   10   4   5   5   4   12
Worker B: 3   4   8   6   7   8   9   10   11   9

1. For worker A, find:
    a. the median
    b. the lower and upper quartiles

2. For worker B, find:
    a. the median
    b. the lower and upper quartiles

3. Draw two box-and-whisker plots representing the workers' times (i.e. one box-and-whisker plot for each worker). Both box-and-whisker plots should be drawn using the same scale.

4. Make one statement comparing the two sets of data. Which worker is the most efficient?

iron = repasser

Exercise 2

In a supermarket, two types of chocolate drops are compared.

The total weight of 20 chocolate drops of brand A is 60.3g.
The mean weight of 30 chocolate drops of brand B is 2.95g.

1. Find the mean weight of a chocolate drop of brand A.
2. Find the mean weight of all the 50 chocolate drops.

STATISTICS

D4 - 91

Recap: You can use your calculator to find the correlation coefficient: it permits you to justify the correlation between two variables. The calculator also gives the least squares regression line in statistics. It is the line of best fit which helps you to make predictions.
For a continuous data, you obtain the median thanks to a cumulative frequency curve: it is the x-coordinate of the point whose y-coordinate is half the total frequency.

Exercise:
1st part: The following scatter graph shows the time spent and the number of books borrowed by 6 people in a library.
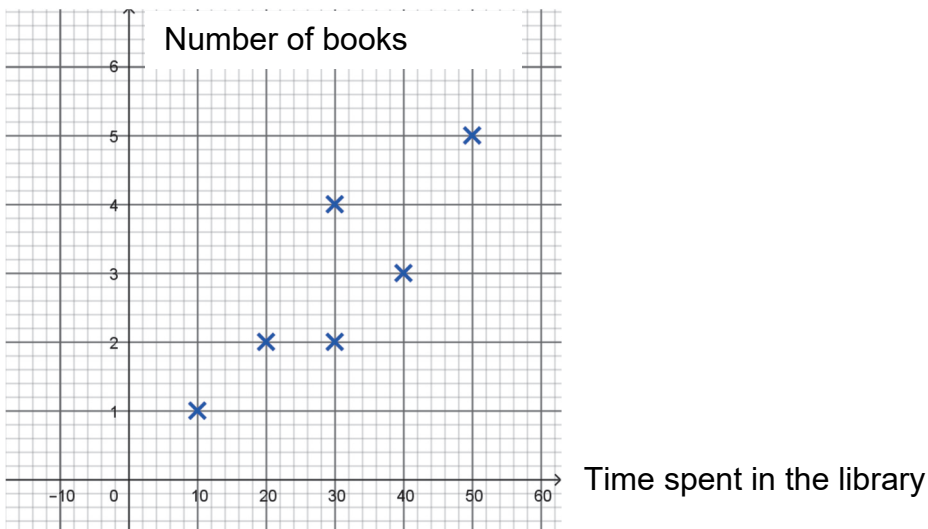
1) Describe the correlation shown and describe the relationship between the time spent in the library and the number of books borrowed.
2) Find an equation of the line of best fit and explain how you could draw it on the scatter graph.
3) Estimate the number of books borrowed if the time spent is 35 minutes.
4) It is **not** sensible to use your line of best fit to estimate the number of books borrowed if a person spends 2 hours in the library. Explain why.

2nd part: A student recorded the time, in minutes, that 40 people spent in the library.

Time t (minutes)	$0 < t \le 15$	$15 < t \le 30$	$30 < t \le 45$	$45 < t \le 60$
Frequency	3	10	20	7

5) Calculate an estimate of the mean number of minutes spent in the library.
6) Explain the method to draw the cumulative frequency curve and draw it on your draft paper.
7) Find an estimate of the median and explain what it means.
8) Explain how you could draw a box plot to show the time spent by these 40 people in the library.

ADVANCED GEOMETRY
Sujet D5 - 52

The first part of this page is a summary which can help you solve the following exercise.

An ordered pair (a,b) of real numbers is a **solution to an inequality in x and y** if the substitution $x = a$ and $y = b$ satisfies the inequality. For example, the ordered pair (2, 5) is a solution of $y < 2x + 3$ because $5 < 2 \times 2 + 3$, that is to say 5 < 7. The graph of an inequality involving two variables is a region of the coordinate plane. The point (2, 7) is on the graph of the line $y = 2x + 3$ but is not a solution of $y < 2x + 3$. A point (2, y) below the line $y = 2x + 3$ is a solution of $y < 2x + 3$.

Exercise:
M. Gable builds models of famous buildings with matches. He needs 5 days and 300 thousand matches to build an Eiffel Tower and 3 days and 450 thousand matches to build a Big Ben Tower. M. Gable has 6 million matches in stock for his models and 60 days to build them.

An Eiffel Tower is sold for 50 pounds and a Big Ben Tower is sold for 70 pounds.

The goal of this exercise is to find how many models of each type M. Gable should sell in order to maximize his **turnover**[1] under the previous constraints.

Let x be the number of Eiffel Tower models built.
Let y be the number of Big Ben Tower models built.
Let T be the turnover in pounds for the sale of x Eiffel Towers and y Big Ben Towers.

Show that $2x + 3y \leq 40$

1) Find another inequality in x and y.

2) Using the following figure, shade the unwanted region corresponding to these inequalities.

3) Show that $y = -\frac{5}{7}x + \frac{T}{70}$.

4) Draw line Δ corresponding to a turnover of 700 pounds.

5) Conclude using the graph.
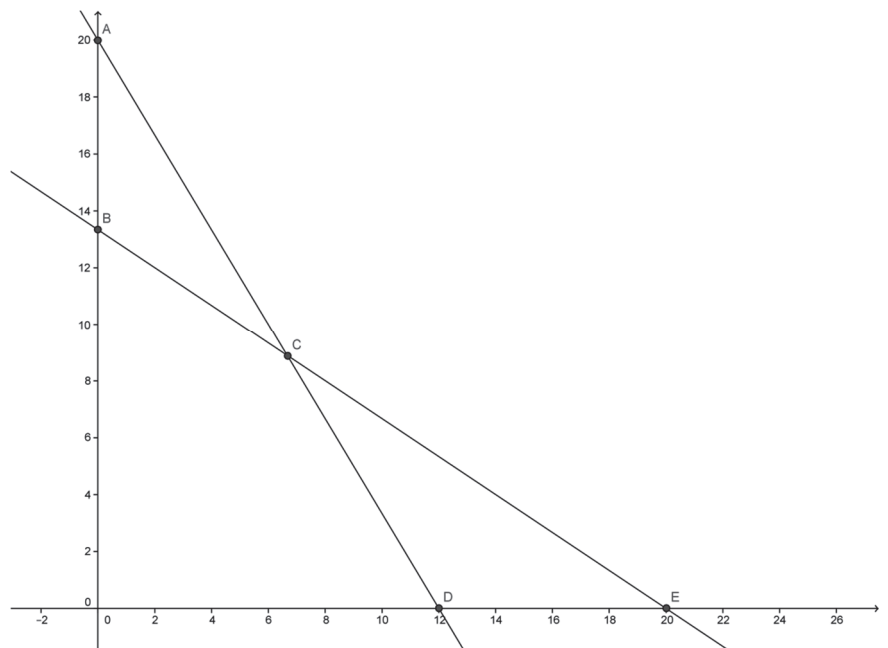
[1] Turnover: *Chiffre d'affaire*

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ADVANCED GEOMETRY

Sujet D5 - 61

The first part of this page is a summary that can be helpful to do the exercise.

An ordered pair (a,b) of real numbers is a **solution to an inequality in x and y** if the substitution $x = a$ and $y = b$ satisfies the inequality. For example, the ordered pair (2, 5) is a solution of $y < 2x + 3$ because $5 < 2 \times 2 + 3$, that is to say 5 < 7. The graph of an inequality involving two variables is a region of the coordinate plane. The point (2, 7) is on the graph of the line $y = 2x + 3$ but is not a solution of $y < 2x + 3$. A point (2, y) below the line $y = 2x + 3$ is a solution of $y < 2x + 3$.

A shop stocks only sofas and beds.
A sofa takes up 3 m² of floor area and is worth £600. A bed takes up 4 m² of floor area and is worth £300.
The shop has 45 m² of floor space to stock.
The shop stocks at least 3 sofas and 2 beds at any one time.
The insurance policy will allow a total of only £6,000 of stock to be in the shop at any one time.
The shop stocks x beds and y sofas.

1. Explain why $x + 2y \leq 20$.

2. Give three other inequalities in *x* and/or *y* to show this information.

3. Could the shop stock 7 beds and 6 sofas at any one time? Explain your answer using your inequalities.

4. Using the diagram:
   a. Shade the <u>unwanted</u> region.
   b. Tell if it's possible for the shop to stock at any one time:
      (i) 4 beds and 4 sofas;
      (ii) 3 beds and 9 sofas.

5. Calculate the coordinates of A.
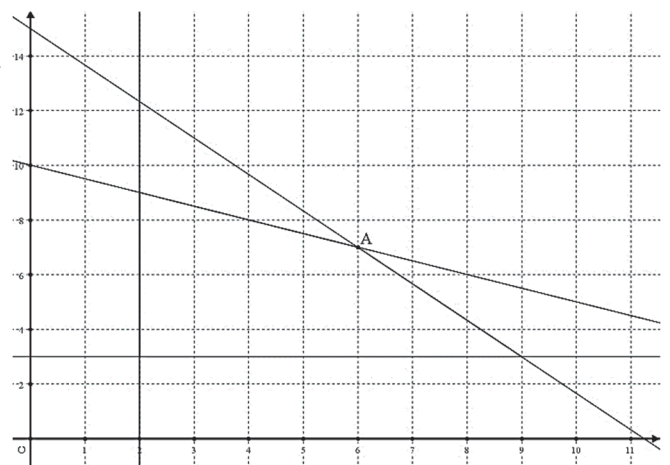
Check your result graphically.

6. A bed is sold £800 and a sofa £1,000.
   a. Determine the equation of a line D corresponding to a revenue of £10,000 and draw the line on the graph.
   b. Is it possible for the shop to make £10,000? What is the maximum revenue the shop can make? Give the corresponding numbers of beds and sofas which have to be sold to make this revenue.

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Sujet D5 – 81

Advanced Geometry

The first part of this page is a summary which may help you solve the following exercise.

In all cases (x_1, y_1) and (x_2, y_2) represent points.

- Midpoint of a line segment: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

- Distance d between two points: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

- Slope m of a line, given two points: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Exercise

An undersea tunnel is to be built from Wicklow, Ireland, to Morfa, Wales, as shown on the diagram. The tunnel will be 45 m below sea level at P but will gradually rise 5 m at point Q. The point S is 4.5 km from Wicklow and the point R is 4.8 km from Morfa. The distance from Wicklow to Morfa is 92.8 km.
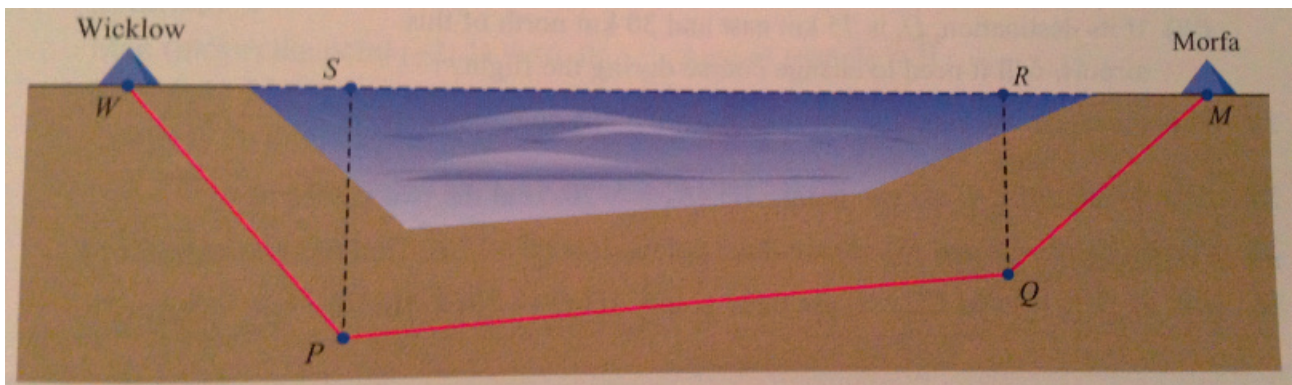
Taking W as the origin $(0,0)$ and using metres as units, find:

(i) the slope of WP, the slope of PQ and the slope of QM ;

(ii) the equation of the line representing the tunnel from P to Q ;

(iii) the total length of the tunnel from W to M, rounding to the nearest metre.

ADVANCED GEOMETRY
Sujet D5 – 82

An ordered pair (a, b) of real numbers is a **solution to an inequality in x and y** if the substitution $x = a$ and $y = b$ satisfies the inequality. For example, the ordered pair (2, 5) is a solution of $y < 2x + 3$ because $5 < 2 \times 2 + 3$, that is to say 5 < 7. The graph of an inequality involving two variables is a region of the coordinate plane. The point (2, 7) is on the graph of the line $y = 2x + 3$ but is not a solution of $y < 2x + 3$. A point (2, y) below the line $y = 2x + 3$ is a solution of $y < 2x + 3$.

Exercise: The Tealicious company ships two types of containers on the Thames, containers with black tea and containers with green tea, under the following constraints:

- The weekly tea production of the company cannot exceed 1200 containers.
- Projections indicate an expected demand of at least 200 containers of black tea per week.
- The demand also requires the shipping of at least two containers of green tea for each container of black tea shipped.

The shipping of a container of black tea yields a profit of £30 whereas the shipping of a container of green tea yields a profit of £20.

Let x be the number of black tea containers shipped per week.
Let y be the number of green tea containers shipped per week.

1- Find three inequalities that must be satisfied by x and y.
2- Match each inequality from the first question to a line of the following graph and shade the region that <u>does not satisfy</u> all the constraints of the Tealicious company
3- Can the Tealicious company ship 400 containers of black tea and 600 containers of green tea?
4- Given that P is the profit for the shipping of x containers of black tea and y containers of green tea show that P satisfies:

$$y = -\frac{3}{2}x + \frac{P}{20}$$

5- Explain how to find the number of containers of each type that must be shipped every week in order to maximize the profit of the Tealicious company and calculate that maximum profit.
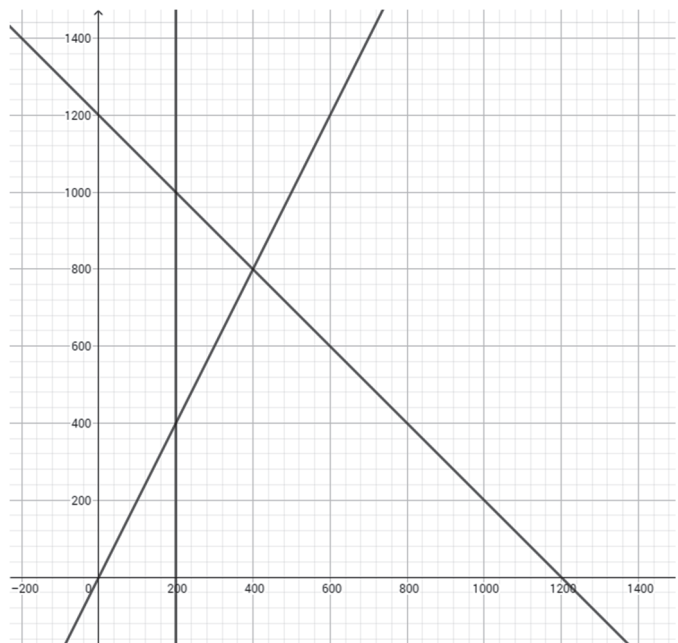
BACCALAURÉATS GÉNÉRAL ET TECHNOLOGIQUE
SESSION 2019

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »
Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Advanced Geometry
Sujet D5 – 83

Bearings:

A bearing is an angle, measured clockwise from the north direction. The bearing of B from A is 80 degrees (note 3 figures are always given) as shown in the diagram on the right.
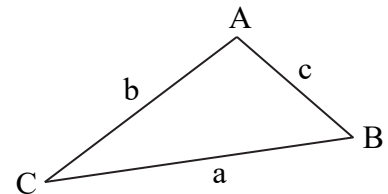
Solving triangles:

In a right-angled triangle, trigonometric ratios and Pythagoras' theorem can be used. Otherwise, triangles may be solved by using the following formulae:

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$ called the *sine rule*;

$$a^2 = b^2 + c^2 - 2bc \times \cos \angle A$$ called the *cosine rule*.

Vocabulary:

One radius – two radii

Due West = Heading to the West

Exercise: On the radar (page 2), we can see two airplanes A and B flying from and to Heathrow (H), one of the airports of London.

1. Explain how to find the bearing of airplane A from Heathrow.

2. Deduce the bearing of Heathrow from airplane A.

3. Find the distance between airplane A and airplane B rounding to the nearest mile.

4. An airplane C leaves Heathrow and flies for 45 minutes at an average speed of 120mph on a bearing of 320 degrees.

    a. Plot the point T where the airplane turns on the radar.

    b. Then it turns due West and continues another 45 minutes at the same speed. How far is airplane C from Heathrow?

    c. What is the bearing of airplane C from Heathrow?

This radar is made of lines radiating at 10° intervals and of concentric circles with centre H. The radius of each circle is 10 miles greater than the former one. The radius of the smallest circle is 10 miles.
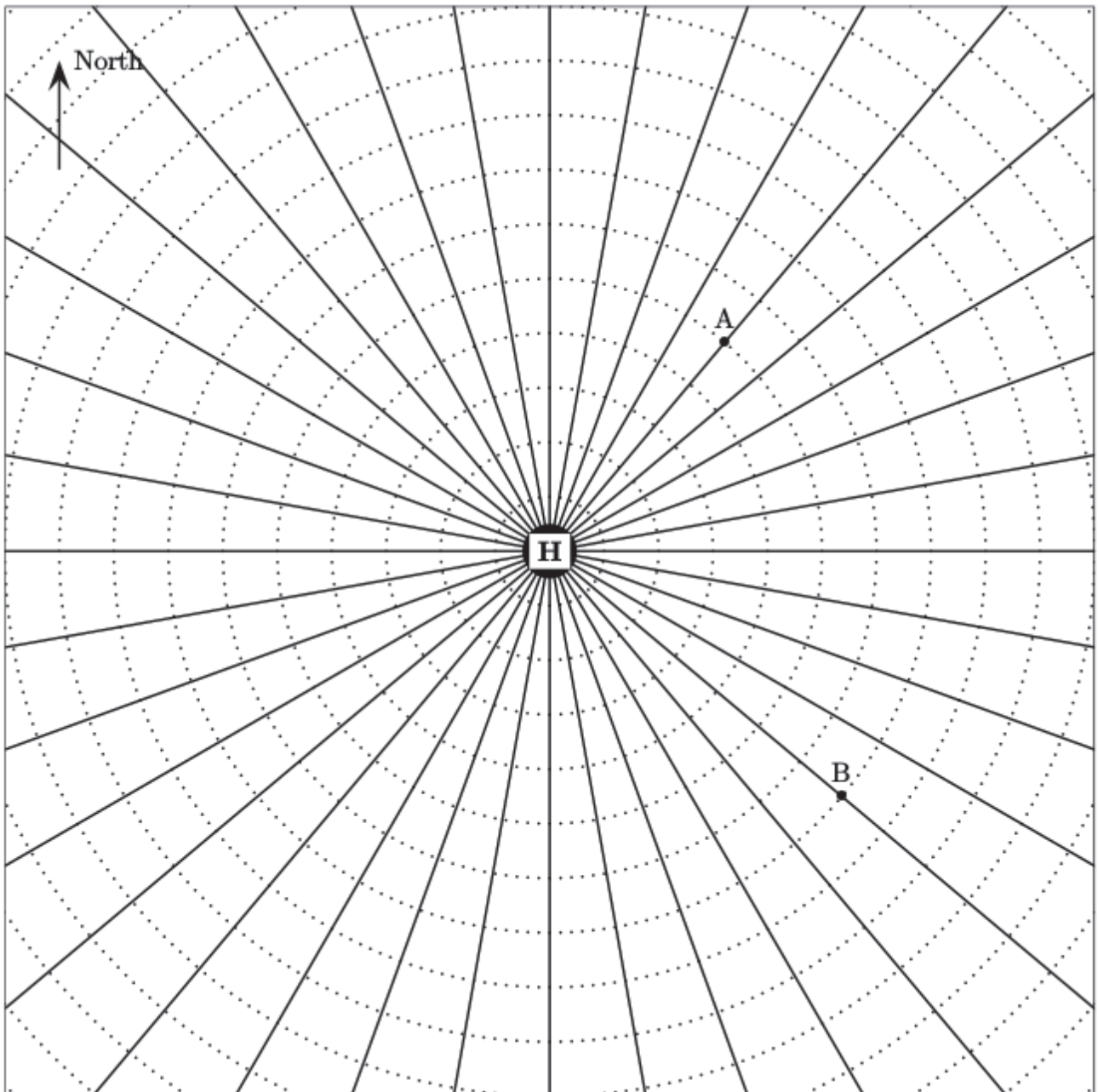
ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »
Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet D5 – 84

ADVANCED GEOMETRY

*The first part of this page is a summary which may help you solve the following exercise.*

An ordered pair (a,b) of real numbers is a **solution to an inequality in x and y** if the substitution $x = a$ and $y = b$ satisfies the inequality. For example, the ordered pair (2, 5) is a solution of $y < 2x + 3$ because $5 < 2 \times 2 + 3$, that is to say 5 < 7. The graph of an inequality involving two variables is a region of the coordinate plane. The point (2, 7) is on the graph of the line $y = 2x + 3$ but is not a solution of $y < 2x + 3$. A point (2, y) below the line $y = 2x + 3$ is a solution of $y < 2x + 3$.

Exercise:

Bob's factory produces two types of pizzas: *"xtra pizza"* and *"yankee pizza"*.
Let x be the number of xtra pizzas which are made.
Let y be the number of yankee pizzas which are made.
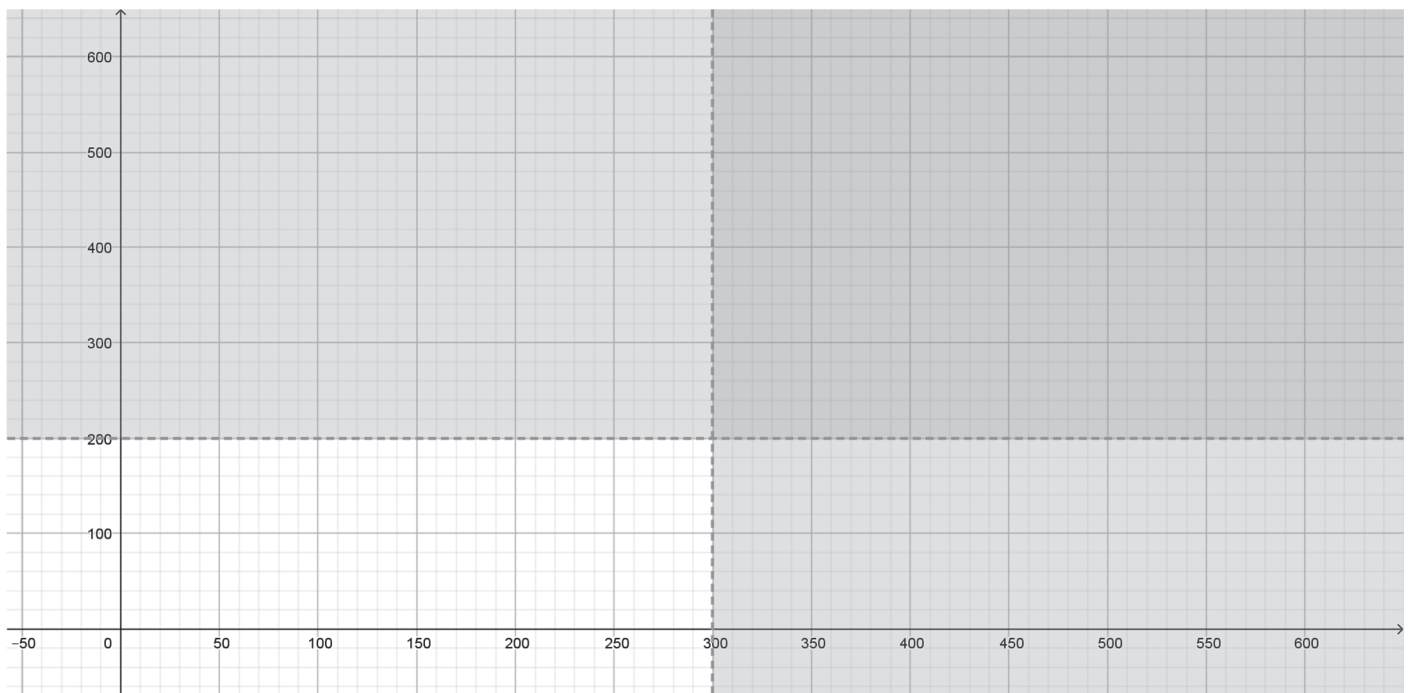
1. Bob's production is subject to different constraints.

    a. Two constraints are $y \leq 200$ and $x \leq 300$. Explain what this means in terms of pizzas.

    b. Explain which of the different regions of the above graph is the accepted one.

2. Each *xtra pizza* requires 5 mushrooms and 8 olives.

Each *yankee pizza* requires 10 mushrooms and 4 olives.

Bob's factory has to use at least 2500 mushrooms and at least 2400 olives.

   a. Write two further inequalities to represent this information.

   b. Add two lines and shadings on the diagram above to represent these two inequalities.

   c. Determine the feasible region and label it $\mathcal{R}$.

3. The factory needs 10 minutes to produce each *xtra pizza* and 4 minutes to produce each *yankee pizza*. Bob wishes to minimize the total time T taken to produce the pizzas.

   a. Write T in terms of x and y, then y in terms of x and T.

   b. Find the optimal number of each type of pizzas that Bob should produce, and find the total time it will take.

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Sujet D5 – 85
Advanced Geometry

Bearings:
A bearing is an angle, measured clockwise
from the north direction. The bearing of B from
A is 80 degrees (note 3 figures are always
given) as shown in the diagram on the right.
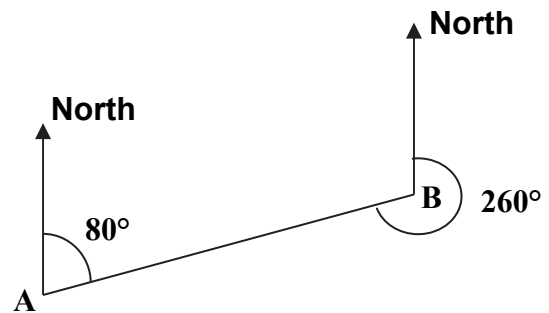
Solving triangles:
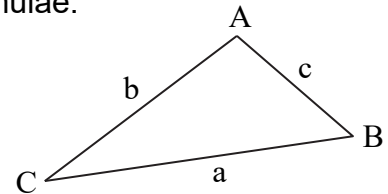In a right-angled triangle, trigonometric ratios
and Pythagoras' theorem can be used.
Otherwise, triangles may be solved by using the following formulae:

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$ called the *sine rule*;

$$a^2 = b^2 + c^2 - 2bc \times \cos \angle A$$ called the *cosine rule*.

A captain wants to sail his ship from port A to port B, but the journey cannot be made
directly. Port B is 40 km North of A.
The ship sails 20 km on a bearing of 075° to reach point P. Then it sails 20 km on a
bearing of 350° and it drops anchor in point S.

1. Plot points P and S on the
   given diagram.
2. Let A be the origin of an
   orthonormal coordinate plane 1
   km unit.
3. Give the coordinates of points
   A and B.
4. Work out the coordinates of
   points P and S. Give values to
   1 d.p.
5. Compute the distance between
   S and B and give the result to
   1 d.p.
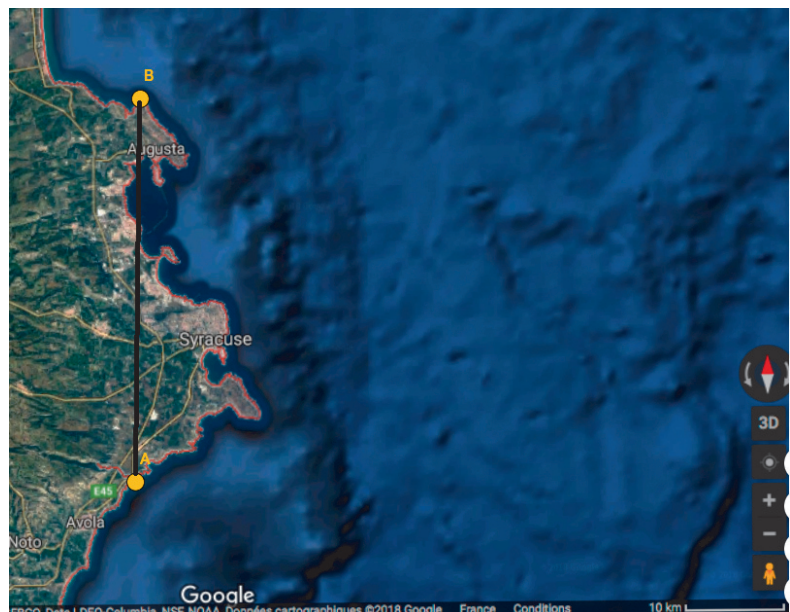
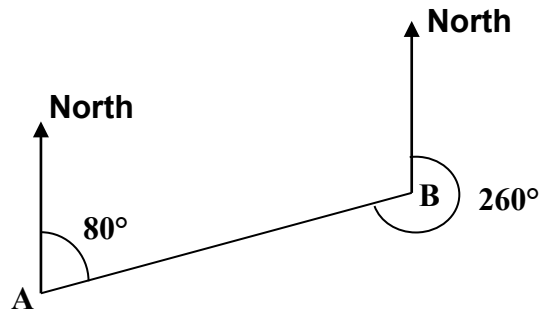**ADVANCED GEOMETRY**

Sujet D5-91

Bearings:

A bearing is an angle, measured clockwise from the north direction. The bearing of B from A is 80 degrees (note 3 figures are always given) as shown in the diagram on the right.
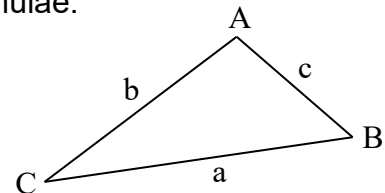

Solving triangles:

In a right-angled triangle, trigonometric ratios and Pythagoras' theorem can be used. Otherwise, triangles may be solved by using the following formulae:

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$ called the *sine rule*;

$$a^2 = b^2 + c^2 - 2bc \times \cos \angle A$$ called the *cosine rule*.

You have been given a map of the local area of Danbury, CT (USA) - page 2 – with two points plotted:

Point A: School Crossroads – where Leyes Lane meets Windy Arbour.

Point B: Glasshouse Crossroads – where Windy Arbour meets Glasshouse Lane.

Question 1:

You have to prepare directions for a plane model. It will fly from A to B,.

An autopilot model plane is located at point A. What instructions should you give to make it fly to point B? Use the scale on the map to convert distances from centimeters into meters, and use a protractor to measure angles.

Question 2:

A firework is fired somewhere in the middle of Danbury. It is visible from the School Crossroads (A) and from the Glasshouse Crossroads (B).

Both observers made a note of the direction, but couldn't make an accurate estimate of how far away the firework was.

Observer 1:"The firework was on a bearing of 100° from where I stood on the School Crossroads."

Observer 2:"I saw the firework from where I was standing at the Glasshouse Crossroads. It was on a bearing of 14° from me".

Explain how to locate the firework on the map.

Question 3:

Another autopilot model plane was given incorrect instructions and is lost. The instructions it followed are given below. The plane took off from point A.

First instruction: Fly a distance of 450m on a bearing of 080°.

Second instruction: Fly a distance of 1000m on a bearing of 242°.

   a. Use these directions to mark the point P on the map where you expect the plane to have landed.
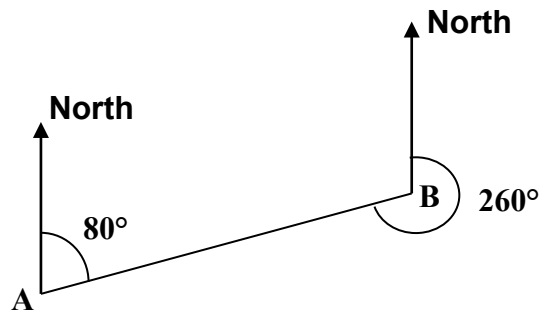   b. Calculate the distance between A and P.

ADVANCED GEOMETRY

Sujet D5-92

The first part of this page is a summary which may help you solve the following exercises.

Bearings:
A bearing is an angle, measured clockwise
from the north direction. The bearing of B from
A is 80 degrees (note 3 figures are always
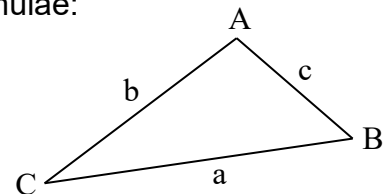given) as shown in the diagram on the right.

Solving triangles:
In a right-angled triangle, trigonometric ratios and Pythagoras' theorem can be used.
Otherwise, triangles may be solved by using the following formulae:

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$ called the *sine rule*;

$$a^2 = b^2 + c^2 - 2bc \times \cos \angle A$$ called the *cosine rule*.

Vocabulary : Due South : heading straight to the South.

Part 1
Matt walks due South from his home. After 4km he changes direction and walks 5 km East.

Calculate the distance between Matt and his home and the bearing of his home from his location.

Part 2
The diagram shows the position of three buildings in a town
The bearing of the church (C) from the school (S) is 056°.
The hospital (H) is due East of the church.
The distance from the church to the school is 3 km and it
is equal to the distance from the church to the hospital.

Work out the bearing of the school from the hospital and
the distance between the school and the hospital.
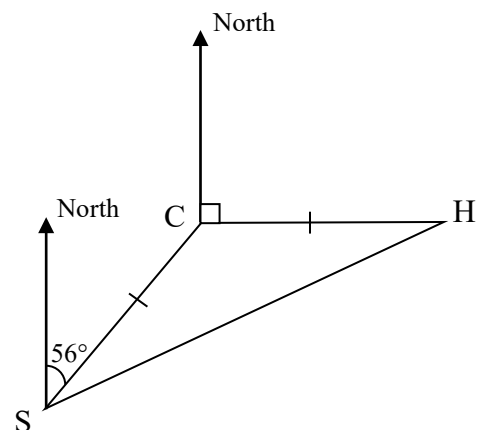
BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
SESSION 2019

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »
Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

PROBABILITY

Sujet D7- 61

Exercise: The Tealicious Company owns jumbo hopper barges and uses them to ship cubic containers on the Thames. Each cubic container can contain either black tea or green tea and can be shipped either to London or to Oxford. The Tealicious Company warehouse contains 6,200 cubic containers, 1,860 of which contain black tea. 20% of the black tea containers should be sent to Oxford whereas 60% of the green tea containers should be sent to London. A cubic container is chosen at random in the warehouse.

Let L be the event "the cubic container is sent to London" and G the event "the cubic container contains green tea".

1.
    a. Show the data in a tree diagram and describe it.
    b. Show that $P(L) = \frac{33}{50}$
    c. After the shipping, a container is picked at random in Oxford. What is the probability that it contains green tea?
2. 10 cubic containers are chosen independently in the warehouse. What is the probability that (give your results rounded to 4 decimal places):
    a. 5 of them should be shipped to London?
    b. At least two of them should be shipped to London?

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »

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Binôme : Anglais / Mathématiques

PROBABILITY

Sujet D7 - 62

The first part of this page is a summary that can be helpful to do the exercise.

Let A and B be two events.

P(A) (respectively P(B)) is the probability that event A (respectively event B) occurs.

$P(A \cap B)$ is the probability that both events A and B occur.

P($A \cup B$) is the probability that at least one of the two events A or B occurs ;
P($A \cup B$)=P(A)+P(B)-P($A \cap B$). If A and B are mutually exclusive, P($A \cup B$)=P(A)+P(B).

P(A)+P(not A) = 1.

P(A|B) (read "the probability of A given B") is a conditional probability. It is the probability that event A occurs given the occurrence of event B.
The multiplication rule states that $P(A \cap B) = P(A|B) \times P(B)$.

On my way to work, I drive through two sets of roadworks with traffic lights which only show green or red. I know that the probability that the first is green is $\frac{1}{3}$. If the first is green, the probability that the second is green is $\frac{1}{3}$. If the first is red, the probability that the second is green is $\frac{2}{3}$.

1) Draw a tree diagram, showing the possible outcomes when passing through both sets of lights.

2) What is the probability of each of the following outcomes?
      a) I do not get held up at either set of lights.
      b) I get held up at exactly one set of lights.
      c) I get held up at least once.
      d) I get held up at the second set of lights.

3) Given that a car has been held up at the second set of lights, what is the probability that it hadn't been held up at the first?

4) Over a school term I make 90 journeys to work. On how many journeys can I expect not to get held up by a red light?

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Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

PROBABILITY

Sujet D7 - 63

The first part of this page is a summary that can help you do the exercise.

Let A and B be two events.
$P(A)$ (respectively $P(B)$) is the probability that event A (respectively event B) occurs.
$P(A')$ is the probability the complement event of A. $P(A') + P(A) = 1$
$P(A \cap B)$ is the probability that both events A and B occur.
$P(A|B)$ (read "the probability of A given B") is a conditional probability. It is the probability that event A occurs given the occurrence of event B.
The multiplication rule states that $P(A \cap B) = P(A|B) \times P(B)$.

Exercise:

In the factory Toys'R'Maths, a machine produces wooden toys for children.
- When the machine operates well, the probability of producing good-quality toys is 0.99;
- When the machine has certain problems, the probability of producing good-quality toys is 0.51;
- Every morning, when the machine is started, the probability that the machine operates well is 0.95.

Let T be the event "the toy produced is a good-quality toy" and N the event "the machine operates well". T' and N' are the complement events of T and N, respectively.

1. Show the information above in a tree diagram.

2. Work out the probability of the event "the machine operates well and produces a good quality toy".

3. a. Explain what the event $T' \cap N'$ is. Calculate $P(T' \cap N')$.
   b. The CEO of Toys'R'Maths states that 97.55% of the production is good-quality toys. Is he right?

4. If on one morning, the first toy produced by the machine is of good quality, what is the probability that the machine operates well?

PROBABILITY

Sujet D7- 71

Prince Charming leaves for a dangerous adventure. He will have to deal with dragons (D) in 20% of cases, trolls (T) in 42% of cases and goblins (G) in every other situation. His probability to win (W) against dragons is 0.4 but he has twice the chance to win against trolls or goblins. If he loses, he dies.

Let W be the event "the prince wins".
T' is the event "not T".

1) Draw a tree diagram and complete it.

2) Explain what the event T' means.

3) What is the probability that he will find goblins and win?

4) What is the probability that he will win the adventure?

5) Given that he has won, show that the probability that he has met a troll is about 0.47 (2 dp).

6) If he wins, the princess will be playing heads and tails to know if she has to accept his proposal. If she has heads, she will say yes, if she has tails she won't. But Prince Charming is smart and he has given her a biased coin so that the probability of having a tail is 0.15. What is the probability that they will get married?

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PROBABILITY
Sujet D7- 72

The first part of this page is a summary that can be helpful to do the exercise.

•Two events A and B are independent if the outcome of one event does not affect the outcome of the other, in which case: P(A and B) = P(A)×P(B).
• P(an event does not occur) = 1 − P(the event does occur).
• A simple tree diagram is a useful way to represent the probabilities of combined events.

EXERCISE

A wheel is divided into twelve equal sectors numbered from 1 to 12.

1. Jack spins the wheel and looks at the number on the sector facing him.
Let A be the event: "He obtains an odd number greater than 4";
Let B be the event: "He obtains a perfect square".
  (i)   Find P(A) and P(B).
  (ii)   Are A and B independent events?

2. Jack spins the wheel twice, independently.
  (i)   Find the probability of the event:
            "The first number he obtains is 9, and the second one is 11".
  (ii)   Find the probability that the sum of the numbers he obtains is 20.
  (iii)   Find the probability he obtains at least one 12.

3. Jack spins the wheel 10 times, independently.
Explain why the probability he obtains at least one 12 is 0.58 to two d.p.

4. How many times *n* should Jack spin the wheel for him to obtain at least one 12, with a probability greater than 99%?

PROBABILITY
Sujet D7- 73

"You want to be careful with these. When they say every flavour, they mean every flavour - you know you get all the ordinary ones like chocolate and peppermint and marmalade, but then you can get spinach and liver and tripe. George reckons he had a bogey-flavoured one once. Ron picked up a green bean, looked at it carefully, and bit into a corner.
"Bleaaargh – see ? Sprouts."

J.K Rowling, Harry Potter and the Sorcerer's Stone, 1997

Harry buys a Bertie Bott's pack of jelly beans. The box contains a total of 40 sweets. Each sweet is either red, green or blue. You can get an ordinary flavor or a weird one.

The number of sweets of each type is shown in the table below:

	red	green	blue
ordinary	8	11	5
weird	6	8	2

1. Harry selects a bean at random, find the probability of it…

   A. …tasting weird.
   B. …not being blue.
   C. …either tasting ordinary or being green.
   D. …tasting weird, given that it's red.

2. Three beans are selected at random, without replacement, hopefully.
Find the probability that two are weird and one is ordinary.

PROBABILITY

Sujet D7 - 91

The first part of this page is a summary that can be helpful to do the exercise.

For two events *A* and *B* the probability of *A* occurring given that B has occurred can be found using the formula $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$.

If *A* and *B* are independent events, *P(A|B)=P(A)* and *P(B|A)=P(B)*.

Mankind is facing a zombie apocalypse. Let's figure out their chances to survive the first month.

70% of people decide to stay at home and wait for the help of authorities. Among these people 1 out of 10 dies (by natural causes or not), 6 out of 10 become a zombie and the others stay alive.

If a person decides to face the danger by going out, he/she has a 30% chance to die, 40% to stay alive, otherwise he/she becomes a zombie.

1) Draw a tree diagram.

2) Find the probability of staying home and surviving.

3) Find the probability of going out and staying alive.

4) Find the probability of turning into a zombie.

5) Find the probability he/she stayed home given that he/she is now a zombie (round your result to 2 d.p.)

6) In this scenario, what would you do and why?

PROBABILITY

Sujet D7 - 92

The first part of this page is a summary that can be helpful to do the exercise.

For two events *A* and *B* the probability of *A* occurring given that B has occurred can be found using the formula *P(A|B)=* $\dfrac{P(A \cap B)}{P(B)}$.

If *A* and *B* are independent events, *P(A|B)=P(A)* and *P(B|A)=P(B)*.

EXERCISE: MARK'S JOURNEY

In this part, results will be given rounded to 2 d.p.

Traffic jams and delayed connections are among the most common causes of missing a flight. Mark is planning a trip from New York to Bordeaux. He books a flight from New York to London Heathrow and a flight from London Gatwick to Bordeaux. He only has a three-hour stopover[1]. Mark is scared to miss the second flight, so he does an internet research and discovers that there is a probability of 0.3 that his first flight is delayed. If the first flight is delayed, he will drive to Gatwick during rush hour which implies that the probability of a traffic jam is 0.6, whereas if the first flight is not delayed the probability of a traffic jam is 0.2.

Let's call D the event "The flight New York – London Heathrow is delayed" and J the event "There is a traffic jam on the way to Gatwick".

1. Give the probability that Mark gets in a traffic jam given that its first flight is delayed.

2. Draw a tree diagram.

3. Compute the probability of the event D∩J.

4. Compute the probability of the event J and justify that events D and J are not independent.

5. Mark finds out that the day of his trip, a huge protest is scheduled on the highway connecting Heathrow and Gatwick. Calculate the probability that the flight will be delayed given that there will be a traffic jam.

a stopover[1] : une escale

PROBABILITY

Sujet D7 - 93

The first part of this page is a summary that can be helpful to do the exercise.

<u>Conditional probability</u>

Let A and B be two events. The probability of A given B is: $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

(if $P(B) \neq 0$).

Law of total probability: if B_1 and B_2 make a partition of the universal set, for any event A of the universal set, $P(A) = P(A \cap B_1) + P(A \cap B_2)$.

<u>Binomial distribution</u>
The expected value, or expectation, of a random variable X following a binomial distribution of parameters n and p is equal to np.

EXERCISE

Around 8% of London's population is over 60.
John Doe is a telesales representative working in London.
The probability he makes a sale on a customer call is 0.4 if the customer is over 60.
Otherwise it is 0.1.

1. John Doe makes a call.
(i)   Draw a tree diagram.
(ii)   Find the probability the customer is over 60 and John makes a sale.
(iii)   Explain why the probability he makes a sale is 0.124.
(iv)   He makes a sale: could you find the probability the customer is over 60?
        Give the result rounded to 3 d.p.

2. John Doe makes a hundred different calls a day over London. Let X be the number of times he makes a sale.
(i)   Explain under which condition(s) X follows a binomial distribution and give its parameters.
(ii)   Find the probability he doesn't make any sale a day.
(iii)   How many sales a day can he expect to make on average?

PROBABILITY

Sujet D7 - 94

The first part of this page is a summary that can be helpful to do the exercise.

A^C denotes the complement of A ; or not A, and $P(A^C) = 1 – P(A)$.

The expected value (or expectation) of a random variable X is denoted by E(X). It is found by multiplying each outcome by its probability and adding the results.

If A and B are two events, then $P(A \text{ and } B) = P(A \cap B) = P(B) \times P(A|B)$
Law of total probability: if B_1 and B_2 make a partition of the universal set, for any event A of the universal set, $P(A) = P(A \cap B_1) + P(A \cap B_2)$.

Exercise 1

A game consists in rolling a fair dice.
If the outcome is 1, you win £1; if the outcome is 2 you win £2 ; for 3, 4, 5 and 6 you win £3, £4, £5 and £6 respectively.
It costs £4 to roll the dice once.
Let *X* be the random variable 'payout of the game'.

a) Copy and complete the following table.

x : payout						
P(*X* = x)						

 b) Find the expected amount you could win or lose if you played that game.
c) Do you think the game is fair? Explain your answer.

Exercise 2

Suppose we send 30 % of our products to company A and 70 % of our products to company B.

Company A reports that 5 % of our products are defective and company B reports that 4 % of our products are defective.

  a) Draw a tree diagram.
  b) Find the probability that a product is sent to company A and it is defective.
  c) Find the probability that a product is sent to company B and it is not defective.
  d) Find the probability that a product is defective.