

**BANQUE DE SUJETS** 

# **ANGLAIS / MATHÉMATIQUES**

SECTION EUROPÉENNE

**SESSION 2018** 

# SOMMAIRE DES ORAUX DE SECTION EUROPÉENNE ou ORIENTALE SESSION 2018

# Anglais / Mathématiques

N°	TITRE DU SUJET
D0_32	Core Knowledge
D0_42	Core Knowledge
D0_46	Core Knowledge
D0_52	Core Knowledge
D0_53	Core Knowledge
D0_62	Core Knowledge
D0_65	Core Knowledge
D0_66	Core Knowledge
D0_68	Core Knowledge
D0_71	Core Knowledge
D0_73	Core Knowledge
D0_74	Core Knowledge
D0_81	Core Knowledge
D0_82	Core Knowledge
D0_83	Core Knowledge
D0_84	Core Knowledge

N°	TITRE DU SUJET
D1_41	Mapping
D1_43	Mapping
D1_52	Mapping
D1_55	Mapping
D1_62	Mapping
D1_63	Mapping
D1_64	Mapping
D1_65	Mapping
D1_66	Mapping
D1_67	Mapping
D1_68	Mapping
D1_73	Mapping
D1_74	Mapping
D1_81	Mapping
D1_82	Mapping
D1_83	Mapping
D1_84	Mapping

N°	TITRE DU SUJET
D2_71	Differentiation
D2_72	Differentiation
D2_81	Differentiation
D2_82	Differentiation

N°	TITRE DU SUJET				
D3_52	Sequences				
D3_62	Sequences				
D3_63	Sequences				
D3_65	Sequences				
D3_71	Sequences				
D3_72	Sequences				
D3_81	Sequences				
D3_82	Sequences				
D3_83	Sequences				
D3_84	Sequences				

N°	TITRE DU SUJET
D4_41	Statistics
D4_42	Statistics
D4_51	Statistics
D4_52	Statistics
D4_53	Statistics
D4_54	Statistics
D4_55	Statistics
D4_61	Statistics
D4_71	Statistics
D4_72	Statistics
D4_81	Statistics
D4_82	Statistics
D4_83	Statistics
D4_84	Statistics

N°	TITRE DU SUJET
D5_41	Advanced Geometry
D5_45	Advanced Geometry
D5_52	Advanced Geometry
D5_61	Advanced Geometry
D5_81	Advanced Geometry
D5_82	Advanced Geometry
D5_83	Advanced Geometry
D5_84	Advanced Geometry
D5_85	Advanced Geometry

N°	TITRE DU SUJET
D7_43	Probability
D7_51	Probability
D7_61	Probability
D7_62	Probability
D7_63	Probability
D7_65	Probability
D7_71	Probability
D7_72	Probability
D7_73	Probability
D7_81	Probability

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE

Sujet D0-32

James Abram Garfield (1831-1881) was sworn in as the twentieth United States president on March 4, 1881. He was the last president to be born in a log cabin (kind of old house made of wood) but he was also the first in some respects:

- he was the first president to have his mother present during his inauguration ceremony;
- he was the first president to be left-handed (actually, he was ambidextrous);
- he was the first (and, so far, the only one) president to develop a proof for the Pythagorean theorem.

Several hundred different proofs of this theorem have been recorded. Still, it is historically interesting that a president of the USA was part of it.

#### First Part

Let's have a look at his proof:

Garfield took a right-angled triangle (number 1) with legs of length a and b and hypotenuse of length c and drew a copy of this same triangle (number 2). Then he drew an additional segment to form a trapezoid, so that a third triangle appears.

Garfield used the fact that the area of a trapezoid is half the product of its height and the sum of the length of its parallel bases.

- 1) Using two different methods, work out the area of the trapezoid, in terms of *a*, *b* and possibly *c*.
- 2) Then explain how Garfield proved the Pythagorean Theorem.

#### Second Part

We know that Garfield was very close to his mother throughout his life, and that she was the first president's mother to attend her son's inauguration ceremony.

Let's imagine that the inauguration hall is composed of a square and an isosceles right-angled triangle as shown in the figure.

Garfield is standing in the triangle and the guests are in the square.

We know that the legs of the triangle measure  $8\sqrt{2}$ m each.

We also know that a guest needs 1.20 m<sup>2</sup> to feel comfortable.

**3)** How many guests can attend the inauguration? Explain your calculations.

# Extra question

You may know the word "trapezium" for "trapezoid", and "Pythagoras' theorem" for "the Pythagorean theorem".

Explain why "trapezoid" and "the Pythagorean theorem" are used in this document.





ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE

Suiet D0-42

The first part of this page is a summary that can help you do the exercise.

Probability is the likelihood of an event to happen. If the outcomes of a random event are equally likely, the probability for any event to occur is:

total number of all possible outcomes

- The perimeter of a triangle with sides *a*, *b* and *c* is: p = a + b + c.
- In a triangle, the "Triangle Inequality" states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side.
- The area of a square whose side's length is c is:  $a = c^2$ .
- In a right isosceles triangle, the hypotenuse's length is  $s\sqrt{2}$ , where s is the length of the legs.



This exercise is a multiple choice test. For each of the four questions, there is one and only one correct answer. Choose the correct answer, and get ready to explain your answer in details.

If you didn't manage to find the answer for a guestion, explain what you tried to do.

#### **Question 1**

A special lottery is to be held to select the student who will live in the only deluxe room in a dormitory. There are 100 seniors, 150 juniors, and 200 sophomores who applied. Each senior's name is placed in the lottery 3 times; each junior's name, 2 times; and each sophomore's name, 1 time. What is the probability that a senior's name will be chosen?

(C)  $\frac{2}{7}$  (D)  $\frac{3}{8}$  (E)  $\frac{1}{2}$ (A)  $\frac{1}{2}$ (B)  $\frac{2}{9}$ 

#### Question 2

If two sides of a triangle have lengths 5 and 6, the perimeter of the triangle could be which of the following? (A) 11 (B) 24 (C) 15

#### **Question 3**

The projected sales volume of a video game cartridge is given by the function  $s(p) = \frac{3000}{2p+a}$  where s is the number of cartridges sold, in thousands; p is the price per cartridge, in dollars; and a is a constant.

If, according to the projections, 100,000 cartridges are sold at \$10 per cartridge, how many cartridges will be sold at \$20 per cartridge? (E) 200,000

(D) 150,000 (A) 20,000 (B) 50,000 (C) 60,000

#### **Question 4**

The diagram shows three squares. The medium square joins the midpoints of the large square. The small square joins the midpoints of the medium square. The area of the small square in the figure is 6 cm<sup>2</sup>. What is the difference between the area of the large square and the area of the medium square, in cm<sup>2</sup>?

(A)  $\sqrt{6}$ (B) 2√<u>6</u> (C) 12 (D) 24 (E) Another answer

Source: SAT Sample Questions http://sat.collegeboard.org

Thales Foundation – Cyprus, Math Kangourou 2011-2013 (Level 7-8 and 9-10)

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE

Sujet D0 - 46

When finding probabilities a tree diagram can be used to list the sample space.



The net of a cuboctahedron is given below. It is composed of 6 squares and 8 equilateral triangles.

- Let's assume that there is the same probability of landing on any face whatever its shape.
   If this object is thrown, what do you think will be the probability of it landing on:
  - a) one of its square faces?
  - b) one of its triangular faces?
- **2.** This cuboctahedron is thrown twice.
  - a) What is the probability of obtaining a triangular face, then a square face?
    - b) What is the probability of obtaining two square faces?
    - c) What is the probability of obtaining two faces of different shapes?



#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE

Sujet D0 - 52

#### Recap:

Imperial units: 1 foot is roughly 30 cm, 1 gallon roughly 4.5 litres. Volume of a cylinder:  $\pi \times r^2 \times h$ , with *r* the radius of the base circle and *h* the height of the cylinder.

#### Exercise:

The paintwork has to be redone in Debbie's bedroom.

His rectangular bedroom is 3 m wide, 4 m long and its walls are  $2\frac{1}{2}$  m high.

- **1)** At first, Debbie wants to know the surface area of all the walls, door and window included. Help her.
- 2) Knowing that the door is 3 feet wide and 7 feet high, and knowing that the window is a square whose side measures 6 feet, calculate the surface area that Debbie has to repaint.
- 3) At the store, Debbie discovers that 1litre of paint is necessary to cover a surface area of 2.5 m<sup>2</sup>. How much paint does she need?
- 4) Debbie has the choice between two different kinds of paint pots: one which contains 3 litres and costs £18, and one which contains 1 gallon and costs £20. Which is the more economical?
- **5)** Finally, she decides to choose the one-gallon paint pots. These are cylindrical. Their height is 30 cm.

Debbie wants to transport the pots in a box whose dimensions are  $15 \text{ cm} \times 45 \text{ cm} \times 30 \text{ cm}$  (width × length × height). Can she?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE

Sujet D0 – 53

**Recap:** We can estimate the median from the graph by reading off the half-way value on the vertical axis. The lower quartile (LQ) is the value one-quarter of the way into the distribution.

The upper quartile (UQ) is the value three-quarters of the way into the distribution.

In a histogram, the frequency of each class is represented by the area of the corresponding bar.

**Exercise:** The time spent by a class of 26 pupils revising a Math test has been collected in the following table. The graph of the corresponding cumulative frequency curve is also shown.



1)

- a. Calculate the mean time spent revising the test by this class.
- **b.** Estimate graphically the number of pupils who revised between 35 and 55 minutes.
- **c.** Estimate graphically the median time, the lower quartile and the upper quartile. Comment on your answers.
- d. Calculate the interquartile range. Comment on your answer.
- 2) The teacher would like to draw a histogram to show the data using the following scale:
  - 1 cm to represent 5 minutes on the x-axis
  - 1 cm<sup>2</sup> to represent 1 pupil

She drew a table to show the heights of the bars of the different classes.

Number of minutes	0-10	10-20	20-25	25-30	30-40	40-60
Height of the bar (cm)	x	У	Z	t	u	1

- **a.** Explain why the height of the bar of the last class is 1 cm.
- **b.** Find *x*, *y*, *z*, *t* and *u*.

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE

Sujet D0 - 62

**Exercise:** The Tealicious company owns jumbo hopper barges and uses them to ship cubic containers on the Thames.

Part A: each cubic container has a side of 5 ft.

- 1. What is the volume of a cubic container?
- 2. A jumbo hopper barge is a huge boat that can be considered as a rectangular prism. It is 100 feet long, 40 feet wide, and 12 feet deep.
  - a. Prove that 384 cubic containers are needed to match the same volume as a jumbo hopper barge.
  - b. Can 384 cubic containers fit in a jumbo hopper barge? Why? Show that the greatest number of cubic containers that can be loaded in a jumbo hopper barge is 320.

Part B: each cubic container can contain either black tea or green tea and can be shipped either to London or to Oxford. The Tealicious company warehouse contains 6,200 cubic containers, 1,860 of which containing black tea. 20% of the black tea containers should be sent to Oxford whereas 60% of the green tea containers should be sent to London. A cubic container is chosen at random in the warehouse.

Let L be the event "the cubic container is sent to London" and G the event " the cubic container contains green tea".

- 1. Show the data in a tree diagram and describe it.
- 2. Show that the probability of event L is 0.66.
- 3. Deduce the number of cubic containers shipped to London.
- 4. How many jumbo barges are needed for that shipping?

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE

# Sujet D0 - 65

<u>Recap:</u> 1 litre =  $1 \text{ dm}^3$ ; 1 mile = 1.6 km; 1 gallon = 4.5 litres

Mr. Thomas has to carry a great quantity of cardboard boxes from his house to his daughter's in his trunk.

- 1) He has to carry 28 boxes of the same size: each is 15 cm wide, 20 cm long and 25 cm high. However, the capacity of his trunk is 200 litres. Could he carry all of them in one trip or will he need to make two trips?
- 2) For one trip, he has to travel 12 km. The fuel consumption of his car (an Aston Martin) is about 19.8 mpg (miles per gallon). How much petrol will Mr. Thomas need for one journey there and back?
- 3) The outward journey takes him 15 minutes. Find his average speed in miles per hour.
- **4)** There are two sets of traffic lights on Mr. Thomas's route to his daughter's. If the traffic light is green, he can drive, otherwise, he has to stop.

The probability that he gets stopped at the first set is 0.3.

If he doesn't stop at the first set, then the probability that he has to stop at the second set is 0.4.

On the other hand, if he gets stopped at the 1<sup>st</sup> set, the probability he has to stop at the 2<sup>nd</sup> set is 0.8. Explain how you will draw a tree diagram to show this probability.

What is the probability that he drives past two green lights?

# SESSION 2018

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »

# Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE

Sujet D0 - 66

#### EXERCISE

Pythagoras' theorem states that in any right-angled triangle:  $a^2 + b^2 = c^2$ , where *c* represents the length of the hypotenuse, and *a* and *b* represent the lengths of the other two sides (called legs of the triangle).

A Pythagorean triple is any set of positive integers that satisfy the relationship  $a^2 + b^2 = c^2$ . Famous Pythagorean triples include (3; 4; 5) and (5; 12; 13).

- (*i*) Show that (8; 15; 17) is a Pythagorean triple.
  (*ii*) Could you find *a* and *b* such that (*a*; *b*; 10) is a Pythagorean triple?
- 2. (*i*) If you add up the same positive integer to each term of a Pythagorean triple, do you still have a Pythagorean triple?
  (*ii*) If you multiply each member of a Pythagorean triple by the same positive integer, do you still have a Pythagorean triple?
  (*iii*) How many different Pythagorean triples do you think there are?
- **3.** The seventeenth-century French mathematician Pierre de Fermat stated without a single proof that the equation  $x^n + y^n = z^n$  has no whole number solutions for *n* greater than 2. What became "Fermat's last theorem" baffled the greatest minds on the planet for over three centuries: it has only been proved recently by a British mathematician named Andrew Wiles.

A rational number can be written under the form  $\frac{a}{b}$ , where a and b are integers,

 $b \neq 0$ .

We know now for sure that it is impossible to find *x*, *y*, *z*, positive **integers** such that  $x^3 + y^3 = z^3$  for instance. But do you think you could find *x*, *y*, *z*, positive **rational numbers** such that  $x^3 + y^3 = z^3$ ?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE

Sujet D0 - 68

You may use the following formula: Volume of prism = base area  $\times$  length.

#### EXERCISE:

1. ABCDEF is a right-angled prism.

- One side length of its base is 4 feet and its height is 5 feet. Its volume is 35 ft<sup>2</sup>. a) Calculate the length of AB.
  - b) Calculate the length of BC. Give your answer to the nearest tenth.



- Adam is twice as old as Jane. Charlie is three years younger than Jane. The sum of all their ages is 53. How old is Jane?
- **3.** First three diagrams in a sequence are shown. How many squares will there be in a diagram 120?



Diagram 1







Diagram 3

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE

Sujet D0\_71

Here's an apparent paradox: most Americans have taken high school mathematics, yet a national survey found that 82 percent of adults could not compute the cost of a carpet when told its dimensions and square-yard price. The Organization for Economic Cooperation and Development recently tested adults in 24 countries on basic "numeracy" skills. The United States ended an embarrassing 22nd, behind Estonia and Cyprus. We should be doing better. Is more mathematics the answer? Calculus and higher math have a place, of course, but it's not in most people's everyday lives. What citizens do need is to be comfortable reading graphs and charts and adept at calculating simple figures in their heads, [understanding decimals and ratios].

So what kinds of questions do I ask my students? One exercise focuses on visualizing data. I have the class prepare a report on how many households in the United States have telephones, land and cell. They are told they have to choose one of the following charts to represent the numbers, and defend their choice.



1) Explain the word "paradox" at the beginning of the text. What is the paradox here?

2) a) What do you call the type of representation of data shown in the text? What does it show?

b) "They are told they have to choose one of the following charts to represent the numbers, and defend their choice". Which one would you choose: A or B? Explain why.

3) A rectangular carpet is 7.3 feet by 10 feet. A square yard of this carpet costs £15. How much does this carpet cost?

(Facts: 1 square feet = 0.11 square yard (2 dp), 1 square yard = 9 square feet)

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE

Sujet D0\_73

# <u>Recap:</u>

Two numbers *a* and *b* are in the ratio p:q if and only if  $\frac{a}{p} = \frac{b}{q}$  (with  $p \neq 0$  and  $q \neq 0$ )

Volume of a cylinder:  $\pi \times r^2 \times h$ , with *r* the radius of the base circle and *h* the height of the cylinder.

Volume of a prism: area of the base × height of the prism.

# Exercise:

John is a great cook and he wants to cook apple jam. In the recipe, the weight of apples and sugar is in the ratio 3:2.

- 1) If he wants to get 1 kg of jam, what are the amounts of apples and sugar he needs?
- **2)** John's sister gave him a 1.5kg pack of apples. How much sugar does he need if he wants to use all the apples for his jam?
- 3) John has got two types of pots to put his jam in. The first one is a cylinder and the second one has the shape of a cuboid whose base is a square. Each pot is 15 cm high. The base circle of the cylinder has a diameter of 5cm whereas each side of the square is 4.5 cm. John wants to use the pot which contains the most. Can you help him?
- 4) John has two children, Ben and Ava. They want to taste his jam. Ben, the elder child, begins, but as he is fussy about his food, the probability he likes it is only 0.6. Moreover, Ava, the younger child, is easily influenced.
  If Ben likes the jam, the probability that Ava likes it is 0.8. Otherwise the probability that Ava likes it is 0.5. Draw a tree diagram and find the probability that Ava likes her dad's jam.
- **5)** Last year, John cooked several types of jam. He got 15 pots of orange jam (200g each), 10 pots of pear jam (150g each) and 12 pots of lemon curd (100g each). What is the mean mass of a pot he cooked last year?

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE Sujet D0\_74

Given the radius R of a circle, the area of a circle can be calculated using the formula:  $\pi R^2$ 

Jonathan wants to redesign his rectangular garden. He made a freehand drawing:



In the garden, there will be a square space for vegetables, and a round swimming-pool. Jonathan wants to plant grass everywhere else.

The diameter of the pool is 3.5 meters Each side of the "vegetables square" is 5 meter long.

The garden is 34 meter long and 15 meter wide.

In his favorite shop, he saw these ads :



1) Find out how much Jonathan should spend to redesign his garden (for  $\pi$  use 3.14. All results should be rounded to 2d.p.).

Be ready to describe each step of your reasoning.

2) Gary's garden is a rectangle as well. Its perimeter P is the same as Jonathan's.

- a) Find a value for P.
- b) Do both gardens have the same area?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

# Binôme : Anglais / Mathématiques

#### CORE KNOWLEDGE Sujet n° D0 81

Recap:

Two numbers *a* and *b* are in the ratio p:q if and only if  $\frac{a}{p} = \frac{b}{q}$  (with  $p \neq 0$  and  $q \neq 0$ )

Volume of a cylinder:  $\pi \times r^2 \times h$ , with *r* the radius of the base circle and *h* the height of the cylinder.

Volume of a prism: area of the base × height of the prism.

A cuboid is a prism whose base is a rectangle.

#### Exercise:

Sandy loves making cupcakes and her friends know it. She owns different cake tins and uses them according to her needs.

- 1) Volumes of the different cake tins:
  - a) For Christmas, the cupcakes cooked by Sandy are cylinders. The cake tins have a base diameter of 5cm and they are 4 cm high. What's the volume of each tin?
  - b) For a dinner, Sandy uses cuboid tins. They are 4 cm high and their base is a rectangle, 5 cm long and 3 cm wide. What's the volume of this tin?
  - c) For a child's birthday, she prefers cooking cupcakes with special tins: triangle based prisms. The base is an isosceles right-angled triangle. Its hypotenuse is  $\sqrt{50}$  cm.

Work out the length of the sides of this triangle, then the volume of this tin given that it is 4 cm high.

# 2) Ratio cake:cream

Of course, at the top of each cupcake, Sandy adds a lovely hat of cream. She estimates that a cupcake is perfect if the ratio of the volumes of cake to cream is 5:3.

For a 60 cm<sup>3</sup> cake, calculate the corresponding volume of cream.

# 3) <u>A particular dinner:</u>

To celebrate her friend's A-Levels, Sandy decides to organize a dinner. She wants to bake 20 cupcakes. Knowing that she needs 2 eggs for a volume of 96 cm<sup>3</sup> (cake and cream altogether), how many eggs does she need?

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE

Sujet D0 82

Unit conversions:		Abbreviations :
1 foot = 30.48 centimetres		1 foot = 1 ft.
1 foot = 12 inches		1 inch = 1" or 1 in.
Area of a disc (given a radius r):	π <b>r</b> ²	

Darts is the sport in which small missiles, called "darts", are thrown at a circular dartboard fixed to a wall.



Dartboard

Darts

The throw line, called oche (say "oki"), is the line behind which the throwing player must stand. The centre of the dartboard is called *centre bull* or *bull's eye* or *bull*. It scores 50 points.

The distance from the centre of the bull to the oche is 9 feet and 7  $\frac{1}{2}$  inches. The distance between the wall and the oche is 7 feet and 9  $\frac{1}{4}$  inches.



- 1) a) Convert 1 inch into centimetres.
  - b) Convert the given distances into centimetres (round to one decimal place).
- 2) a) Given that the wall is perpendicular to the floor, work out the distance between the floor and the center of the bull (round to the nearest cm).
  - b) The real distance is 172.72 cm. Convert it into feet and inches.

3) A player randomly throws a dart (and the dart hits the dartboard). The diameter of the dartboard is 34 cm. The diameter of the bull is 1.4 cm. What is the probability that the dart hits the bull? (to 4 dp)

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles Binôme : Anglais / Mathématiques

#### CORE KNOWLEDGE

# Sujet D0\_83

**Document 1**: London 2017 World Athletics Championships



IAAF World Championships Marathon, Sunday 6 August 2017 For course details & Championships info visit Iondon2017athletics.com

#### 

Set against some of the capital's most historically-significant and picturesque backdrops, the 26.2 mile marathon route will start and finish at Tower Bridge in central London. The route will comprise four laps of a 10 km course on closed roads, heading west along Victoria Embankment towards the Houses of Parliament, then back alongside the River Thames to St Paul's Cathedral, and returning to the Tower of London.

Source : http://www.london2017athletics.com

#### Questions:

- 1. Present 2017 IAAF world championships marathon: total length of the course, length of one lap, length between the start and finish points and the lap turn point. Give all your answers in miles and in kilometers.
- 2. During the 26.2 mile men's marathon (rounded to the nearest tenth) at the 2012 Summer Olympics:
  - 85 runners finished the race. The winner in 2:08:01 and the last runner in 2:55:54.
  - 20 other runners did not finish the race.

Give an interval for the total amount of miles run by all the participants together. Present your results in standard form, to 2 significant figures.

If all the 85 participants who finished the race had done it one after the other, how many days would it have taken?

#### Notes:

Backdrop = *décor, arrière-plan* 

1 mile = 1609,34 meters

IAAF = International Association of Athletics Federations

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# CORE KNOWLEDGE Sujet n° D0\_84

Three, One, Four, One, Five, and On The numbers recount their endless tale. Three - Barefoot green, a silent voice. White as hunger, One is twice Bright like babies' eyes. Four is timid, envious of E. Five, Punctuation or a pregnant sigh Precedes proud Nine, colour of falling night. Two, an unfastened knot, A wayward wind, the hollow of Six resounding. Nearby, Eight, a cloud of fireflies above a lake Over which I skim Sevens Remembering that Zero is nothing but a circle.

Pi Poem, Daniel TAMMET, 2009

Daniel Tammet (born January 31,1979) is an English writer, essayist, translator, and autistic scientist. He holds the European record for reciting  $\pi$  from memory to 22,514 digits in five hours and nine minutes on March 14th 2004.

1.

- **a)** Use your calculator to work out  $\pi$ -3.14 and 22/7- $\pi$  correct to 5 dp.
- **b)** Daniel Tammet chose to recite  $\pi$  on Pi Day: March 14<sup>th</sup> (**3**rd month **14**th day). Why do you think some mathematicians believe Pi Day should be July 22nd?

2.

**a)** To this day more than 22 trillion digits of π have been discovered. An average person can read out approximately 120 digits/min.

Keeping this pace, how long would it take to recite these digits?

- **b)** Assuming a total world population of roughly 7 billion people, how many digits of  $\pi$  would everyone have to memorize in order to preserve all known digits of  $\pi$ ?
- **3.** Let's view  $\pi$  as a big, random string of numbers. The odds of finding a string of digits in the first 100 million digits of  $\pi$  are:

String length 1-5		6	7	8	9	10	11
Chances of finding	100 %	nearly 100 %	99.995 %	63 %	9.5 %	0.995 %	0.09995 %

- a) If we search for the digit "6" in  $\pi$ , what is the chance that a digit picked at random in the first 100 million decimals of  $\pi$  is equal to "6"?
- b) If we search for the string of digits "61" in  $\pi$ , what is the chance that a string of two digits picked at random in the first 100 million decimals of  $\pi$  is equal to "61"?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# MAPPING

Sujet D1 - 41

The first part of this page is a summary that can help you do the exercise.

Let *f* be a function. If f(2) = 3, then 3 is called the image of 2 under *f*, and 2 is a pre-image of 3 under *f*. The domain of *f* is the set of all the numbers that have an image under *f*. The range of *f* is the set of all the numbers that can be written as images of a number under *f*.

Chelsea and Henrietta are practicing diving.



When Chelsea jumps off the diving platform, she reaches a maximum height of 9 feet above the water after moving 1 foot in the horizontal direction, and she lands in the water after moving 4 more feet in the horizontal direction. Her height in feet above the water as a function of horizontal distance x in feet can be represented as a quadratic function C(x).

Henrietta just runs straight off another platform without jumping. Her height in feet above the water is given by a function of horizontal distance x in feet,  $H(x) = -\frac{2}{9}x^2 + 8$ .

- **1.** Since Chelsea reaches her maximum height when x = 1, C(x) can be written as  $C(x) = a(x-1)^2 + b$ . Explain how to work out the values of *a* and *b*.
- 2. Who started on a higher platform? Explain your answer.
- **3.** When Henrietta lands in the water, how many feet in the horizontal direction has she moved from the diving platform?
- 4. Give the domain and the range of each function *C* and *H*.
- **5.** The solution of the equation C(x) = H(x) is roughly 3.7. What does it mean for Chelsea and Henrietta?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## MAPPING

#### Sujet D1 - 43

The first part of this page is a summary that can help you do the exercise.

• Quadratic functions are polynomials in which the largest exponent is 2. The graph of a quadratic function is always a parabola, and the general form of the equation is  $y = ax^2 + bx + c$ . If a > 0, the parabola opens up and has a minimum value.

If a < 0, the parabola opens down and has a maximum value.

The *x*-coordinate of the vertex of the parabola is equal to -b/2a, and the axis of symmetry is the vertical line whose equation is x = -b/2a.

• The solutions to any quadratic equation  $ax^2 + bx + c = 0$  such as  $\Delta = b^2 - 4ac > 0$  are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

When Tim shoots an arrow in the air with <u>his bow</u>, the tip of his arrow is 2 metres above the ground. Then it reaches its maximum height after 2 seconds. Its height above the ground in metres is given by the quadratic function h(t) where *t* is the time in seconds from when the arrow is shot.

a) After describing the trajectory of the arrow, explain why the tip of the arrow is again 2 metres above the ground at t = 4 seconds.

b) Given that the tip of the arrow is 20 meters above the ground 3s after the beginning of the shot, show that  $h(t) = -6t^2 + 24t + 2$ .

c) Tim thinks his arrow reached a maximum height of 30 metres. Discuss his hypothesis.

d) How could you find the value of *t* for which the arrow meets the ground?

His bow : son arc.

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# MAPPING

# Sujet D1 – 52

Recap:

A quadratic function f of x is a function in the form  $f(x) = ax^2 + bx + c$  with a, b, c three real numbers and  $a \neq 0$ .

The graph of this function is a parabola, a line symmetric curve,

and the *x*-coordinate of its vertex is  $x = -\frac{b}{2a}$ .

Romeo and Juliet

It's May 4<sup>th</sup> 2015 at night. Juliet is in her bedroom with the window half-open. Romeo wants to leave her a message but his cell phone is out of order. He writes his message on a piece of paper and wraps it around a stone that he tries to throw through her window.

Juliet's window is at a height of 3.80m and it is 1 m high.

It is a sash-window<sup>1</sup> and its half-top is closed whereas its half-bottom is open.

Romeo is 3 m away from Juliet's wall.

Romeo throws the stone. The height h of the stone is a quadratic function of x, x being the horizontal distance from Romeo after beginning the throw.

At the beginning of the throw, the stone is 1 m high.

Then, when x = 1, the stone is 6 m high,

and when x = 2, the stone is 7 m high.

- **1)** Give three values of (x, h).
- **2)** Prove that  $h(x) = -2x^2 + 7x + 1$ .
- 3) What could you say about the trajectory of the stone?
- **4)** Do you think that the stone with the message will reach Juliet?

Can it reach Juliet without her window being broken?



<sup>1</sup> sash-window: *fenêtre à guillotine* 

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# MAPPING

# Sujet D1 – 55

**Quadratics:** the roots of the quadratic equation  $a x^2 + b x + c = 0$ , where,  $a \neq 0$ , are given by the formula  $x = \frac{-b \pm \sqrt{\Delta}}{2a}$ , if the discriminant  $\Delta = b^2 - 4 ac$  is a positive real number. **Formulas:** area of a circle with a radius of  $r : \pi \times r^2$ **Ratios:** two numbers *a* and *b* are in the ratio n : m if  $\frac{a}{n} = \frac{b}{m}$ , where *n* and *m* are two integers different from 0.

Pizzas often come in three sizes: large, medium and individual. When you change the size of the pizza, how does the amount of cheesy, delicious topping compare to the amount of crust...

1) Calculate the total area for each pizza: large, medium and individual. Then, calculate how much of the total area is the cheesy, delicious topping part, and how much is crust. Which size would you prefer and why?



- 2) Most pizzas have a 1-inch wide crust. Based on this
  - a) Write functions for the total area, the inside area, and the crust area in terms of the radius, r.
  - b) For which size pizza is the inside area equal to the crust area... and would you order it?
- **3)** a) Calculate the ratio of crust area to pizza area for the 7-inch radius pizza.
  - b) Which size of pizza should you order if you want the ratio of crust area to pizza area to be 1:3?

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

## Binôme : Anglais / Mathématiques

#### MAPPING

#### D1- 62

The first part of this page is a summary that can help you do the exercise.

Quadratic functions are polynomials in which the largest exponent is 2. The graph is always a parabola, and the general form of the equation is y = ax<sup>2</sup> + bx + c. If a > 0, the parabola opens up and has a minimum value. If a < 0, the parabola opens down and has a maximum value. The x-coordinate of the vertex of the parabola is equal to -<sup>b</sup>/<sub>2a</sub>, and the line of symmetry is the vertical line whose equation is x = -<sup>b</sup>/<sub>2a</sub>.
When b<sup>2</sup> - 4ac > 0, the solutions of the quadratic equation ax<sup>2</sup> + bx + c = 0 (where a ≠ 0) are given by the quadratic formula : x = -<sup>b±√b<sup>2</sup>-4ac}/<sub>2a</sub>. If b<sup>2</sup> - 4ac = 0, the quadratic equation ax<sup>2</sup> + bx + c = 0 (where a ≠ 0) has only one solution : x = -<sup>b</sup>/<sub>2a</sub>. If b<sup>2</sup> - 4ac < 0, the quadratic equation ax<sup>2</sup> + bx + c = 0 (where a ≠ 0) has no real solution.
</sup>

The Fish&Ship Company wants to ship containers of cod fish to London. The cost, in pounds (£), to ship *x* containers is a quadratic function of *x* that is to say that  $C(x) = ax^2 + bx + c$ , where *a*, *b* and *c* are real numbers and  $a \neq 0$ . The fixed costs of the company are £ 2750; the cost to ship 10 containers is £4250 whereas the cost to ship 20 containers is £8550.

1- Show that for all real number  $x, C(x) = 14x^2 + 10x + 2750$ .

The revenue for each container transported is £570.

Let P(x) be the profit of the company for x containers shipped.

- 2- Show that for all real number *x*,  $P(x) = -14x^2 + 560x 2750$ .
- 3- How many containers at least must be shipped to have a positive profit?
- 4- What can you advise the manager of the Fish&Ship Company maximize its profit?
- 5- Describe the graph of *P* in a cartesian coordinate plane.

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# MAPPING

Sujet D1 - 63

# Exercise:

We know that the roots of the quadratic equation  $x^2 + b x + c = 0$ , are given by the formula  $x_1 = \frac{-b + \sqrt{\Delta}}{2}$ , and  $x_2 = \frac{-b - \sqrt{\Delta}}{2}$  if the discriminant  $\Delta = b^2 - 4c$  is a positive real number.

We suppose the discriminant is a positive number.

- 1. Use the quadratic formula to show that  $x_1 + x_2 = -b$  and  $x_1 x_2 = c$ .
- 2. We want to solve  $x^2 + 3x 28 = 0$  without using the quadratic formula. a. Find all pairs of integers  $x_1$  and  $x_2$  such that  $x_1 x_2 = -28$ .
  - b. In the previous pairs find which pair  $x_1$  and  $x_2$  is such that  $x_1 + x_2 = -3$ .
  - c. What are the solutions of  $x^2 + 3x 28 = 0$ ?
  - d. Solve  $x^2 17x + 72 = 0$  with the same method.
  - e. Solve  $x^2 + 13x + 36 = 0$  with the same method.
- 3. Let x and y be two numbers such that their product is -15 and their sum is 2.
  - a. Show that *x* must be a solution of a quadratic equation.
  - b. Solve the quadratic equation with the quadratic formula.
  - c. Find out the values of *x* and *y*.

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

# Binôme : Anglais / Mathématiques

## MAPPING

#### Sujet D1 - 64

The first part of this page is a summary that can help you do the exercise.

Quadratic functions are polynomials in which the largest exponent is 2. The graph is always a parabola, and the general form of the equation is y = ax<sup>2</sup> + bx + c. If a > 0, the parabola opens up and has a minimum value. If a < 0, the parabola opens down and has a maximum value. The x-coordinate of the vertex of the parabola is equal to -<sup>b</sup>/<sub>2a</sub>, and the line of symmetry is the vertical line whose equation is x = -<sup>b</sup>/<sub>2a</sub>.
When b<sup>2</sup> - 4ac > 0, the solutions of the quadratic equation ax<sup>2</sup> + bx + c = 0 (where a ≠ 0) are given by the quadratic formula : x = -<sup>b±√b<sup>2</sup>-4ac}/<sub>2a</sub>. If b<sup>2</sup> - 4ac = 0, the quadratic equation ax<sup>2</sup> + bx + c = 0 (where a ≠ 0) has only one solution : x = -<sup>b</sup>/<sub>2a</sub>. If b<sup>2</sup> - 4ac < 0, the quadratic equation ax<sup>2</sup> + bx + c = 0 (where a ≠ 0) has no real solution.
</sup>

A factory produces umbrellas. It can produce up to 300 umbrellas per week.

The cost to produce umbrellas depends on the number of umbrellas produced. If we label it *x*, the director has to pay a cost in £ equal to  $C(x) = -0.10x^2 + 30x + 1200$ . Each umbrella is sold £18.

**1)** If the factory produces 150 umbrellas, how much does it cost? Does the factory make a profit in that case?

For the following questions, we label *x* the number of umbrellas produced.

- 2) We already know that the expenses are given by the value of C(x). What kind of function is C? Explain how you can draw the curve of function C. For how many umbrellas produced does the factory have the highest expense?
- **3)** Express the receipts, R, in terms of *x*. What kind of function is R? Explain how you can draw the curve of function R.
- **4)** Express the profit P in terms of *x*.
- 5) For how many umbrellas produced does the factory earn money?
- 6) Explain how you can find the highest profit per week and give its value.

## SESSION 2018

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »

# Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# MAPPING

Sujet D1 - 65

The first part of this page is a summary that can help you do the exercise.

Quadratic functions are polynomials in which the largest exponent is 2. The graph is always a parabola, and the general form of the equation is y = ax<sup>2</sup> + bx + c. If a > 0, the parabola opens up and has a minimum value. If a < 0, the parabola opens down and has a maximum value. The x-coordinate of the vertex of the parabola is equal to -<sup>b</sup>/<sub>2a</sub>, and the line of symmetry is the vertical line whose equation is x = -<sup>b</sup>/<sub>2a</sub>.
When b<sup>2</sup> - 4ac > 0, the solutions of the quadratic equation ax<sup>2</sup> + bx + c = 0 (where a ≠ 0) are given by the quadratic formula : x = -<sup>b±√b<sup>2</sup>-4ac}/<sub>2a</sub>. If b<sup>2</sup> - 4ac = 0, the quadratic equation ax<sup>2</sup> + bx + c = 0 (where a ≠ 0) has only one solution : x = -<sup>b</sup>/<sub>2a</sub>.
If b<sup>2</sup> - 4ac < 0, the quadratic equation ax<sup>2</sup> + bx + c = 0 (where a ≠ 0) has no real solution.
</sup>

#### EXERCISE

The height (in metres, from the ground) of a stone launched from a catapult is given by:

$$h(t) = 20t - 9.8t^2$$

where t is the time (in seconds) after the moment of launching.

When the question is about a time, give your answer correct to 1 millisecond (= correct to 3 d.p.).

- 1) Find when the stone hits the ground.
- 2) For how long is the stone higher than 5 metres above the ground?
- 3) Could the stone reach a height of 12 metres above the ground?
- 4) Explain how you can find the maximum height of the stone.
- **5)** Describe the graph of function *h*.

Source: The Centre for Innovation in Mathematics Teaching (CIMT)

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# MAPPING

#### Sujet D1-66

The first part of this page is a summary that can be useful to do the exercise.

A linear function is a function f which can be written in the form f(x) = ax + b. The graph of a linear function is a straight line.

The overheads are also called burden costs, they correspond to the expenses that cannot be avoided.

Aicha is the manager of a company which manufactures calculators. She prepares a report on production costs, expenses and returns.

Each calculator costs the company £15 to produce. In addition, the company has monthly overhead costs of £19,710.

The selling price of each calculator is £45.

- 1. Show that the total cost, *C*, of producing *x* calculators each month, is: C(x) = 15x + 19710.
- **2.** Write a formula describing the selling price, S(x), of x calculators.
- 3. Plot and label the graphs of functions *C* and *S*, for  $0 \le x \le 1000$ .
- **4.** The point of intersection of the two graphs is called the *break-even point*. Explain what this means in terms of the given problem. Find the coordinates of the break-even point (point of intersection).
- **5.** Shade the portion between the two graphs to the left of the break-even point. Explain what this portion represents.

Profit may be defined as the selling price minus the total cost.

- **6.** Write a formula describing the profit obtained, *P*, after selling *x* calculators.
- 7. Determine whether a profit or loss is made when:
  - a. 400 calculators are sold in a particular month;
  - **b.** 800 calculators are sold in a particular month.
     Check your answer with the graph you drew in question 3.

## SESSION 2018

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »

#### Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

#### MAPPING

#### Sujet D1 - 67

#### Summary:

A quadratic polynomial is a polynomial of the form  $ax^2 + bx + c$ . The values of *x* for which a quadratic is equal to zero are called the roots of this quadratic. The number of real roots of a quadratic depends on the sign of  $\Delta = b^2 - 4ac$  which is called the discriminant.

- If  $\Delta > 0$  then the quadratic has two real roots.
- If  $\Delta = 0$  then the quadratic has one real root (two equal real roots).
- If  $\Delta < 0$  then the quadratic has no real root.

The extremum of a quadratic occurs when x is equal to  $-\frac{b}{2a}$ .

#### Exercise :

A ball dropped with no initial velocity falls free.

With a chronophotography, we know how far the ball has fallen down every 20 ms (milliseconds).

We write d(t) the distance in cm the ball has fallen down t ms after it was dropped.

t	0	20	40	60	80	100	120
<i>d</i> ( <i>t</i> )	0	0,2	0,8	1,8	3,2	5	7,2

- **1.** Represent (t, d(t)) in a coordinate system.
- 2.
- **a.** Is *d* a linear function?
- **b.** Can you make any conjecture about the nature of *d*?
- **3.** We want to find *a*, *b*, *c* such as  $d(t) = at^2 + bt + c$ . Use the table above to find the values of *a*, *b* and *c*.
- 4. Does the expression (found in question 3) work for all the values of the table?
- 5. We suppose the expression of *d*(*t*) found in question 3 is valid for any value of *t*. The ball was dropped from a height of 1 m above the ground. When does it reach the floor? Give the time in seconds.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

# MAPPING

#### Sujet D1 - 68

#### Recap

A quadratic polynomial is a polynomial of the form  $ax^2 + bx + c$ . The values of *x* for which a quadratic is equal to zero are called the roots of this quadratic. The number of real roots of a quadratic depends on the sign of  $\Delta = b^2 - 4ac$  which is called the discriminant.

- If  $\Delta > 0$  then the quadratic has two real roots.
- If  $\Delta = 0$  then the quadratic has one real root (two equal real roots).
- If  $\Delta < 0$  then the quadratic has no real root.

The extremum of a quadratic occurs when x is equal to  $-\frac{b}{2a}$ .

The break-even point is the point at which cost and revenue are equal.

A company manufactures and sells *x* radios per month.

The cost, C, in dollars, involved in producing x radios per month is given by the equation

$$C(x) = 60x + 70000 \qquad 0 \le x \le 6000.$$

The revenue, *R*, in dollars, based on the sales of *x* radios per month is given by the equation

$$R(x) = -\frac{1}{30}x^2 + 200x, \qquad 0 \le x \le 6000.$$

- 1. Draw accurately the graphs of the cost and revenue functions on the same set of axes.
- 2. Calculate:
  - a. the minimum cost involved;
  - **b.** the maximum revenue.
- 3. Why is there a cost involved when no radios are produced?
- **4.** On your graph, identify the break-even points.
- 5. What profit does the company make when 2000 radios are produced and sold?
- **6.** a) Find an expression in terms of *x* for the profit, *P*, in dollars, this company makes on the sales of their radios.
  - b) How many radios would they need to sell to earn \$ 60,000?
  - c) How many radios would they need to sell to achieve this maximum profit?
  - d) What is the maximum profit the company can hope to make?
- 7. For what values of *x* will the company be in the red?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

# MAPPING

#### D1 - 73

The first part of this page is a summary that can help you do the exercise.

• Quadratic functions are polynomials in which the largest exponent is 2. The graph of a quadratic function is a parabola, and its equation is  $y = ax^2 + bx + c$ .

If a > 0, the parabola opens up and has a minimum value. If a < 0, the parabola opens down and has a maximum value. The *x*-coordinate of the vertex of the parabola is equal to  $\frac{-b}{2a}$ , and the axis of symmetry is the vertical line whose equation is  $x = \frac{-b}{2a}$ .

• Given the radius *R* of a circle, the circumference of a circle can be calculated using the formula :  $2\pi R$  and the area using the formula :  $\pi R^2$ 

A Norman Window is a window that has the shape of a semicircle on top of a rectangle (the diameter of the circle is equal to the width of the rectangle):



The goal of this problem is to maximize the area of a Norman window with a given perimeter so that it lets in as much light as possible.

- 1) The **perimeter** of the window is **20 feet**. Write *y* in terms of *x* (use 3.14 for  $\pi$ ).
- 2) Show that the area A of the window is equal to :

$$A(x) = -0.8925x^2 + 10x$$

3) Work out the value of *x* so that the area *A*(*x*) is maximum and find the maximum area (rounded to 1d.p.).

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

# MAPPING

D1 - 74

The first part of this page is a summary that can be helpful to do the exercise.

The standard form equation of a quadratic function is: f(x) = ax<sup>2</sup> + bx + c with a, b and c constants, a ≠ 0. The graph of f is called a parabola. The x-coordinate of the vertex of the parabola is - b/2a.
The domain of a function f is the set of all the numbers that have an image under f. The range of a function f is the set of all the numbers that can be written as images of a number under f.
To solve the quadratic equation :ax<sup>2</sup> + bx + c = 0 with a ≠ 0, compute the discriminant Δ= b<sup>2</sup> - 4ac ->If Δ>0, the solutions of the quadratic equation are given by the quadratic formula x = (-b±√b<sup>2</sup>-4ac)/(2a).

If  $\Delta$ =0, there is one real number solution  $x = \frac{-b}{2a}$  called a double root of the equation.

If  $\Delta$ <0, there is no real number solution.

# Exercise

- **1.** For x in [0;6], let f be the function by: f(x) = 2x 1.
  - (*i*) Describe the graph of function *f*.
  - (*ii*) What is the range of function *f*?
- **2.** Let g be the function defined for all x by:  $g(x) = x^2 4x + 3$ .

(i) Solve for x the equation g(x) = -1, then the equation g(x) = 3.

- (*ii*) Describe the graph of function g.
- (*iii*) What is the range of function g?

**3.** Do the curves of equation y = f(x) and y = g(x) intersect? How would you find the coordinates of the intersection points?

**4.** *h* is a quadratic function, whose range is  $[5; +\infty]$ , and whose graph passes through A(1; 7).

- (*i*) In your opinion, how many of such functions are there?
- (ii) Could you find an expression for h(x) in terms of x for at least one such function?

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# MAPPING Sujet n° D1\_81

The first part of this page is a summary that can help you do the exercise.

A function takes one element of a first set ("domain") and assigns it to one, and only one, element of a second set ("range").

The function maps each element of the domain onto its image in the range. If a function can map more than one element of the domain onto the same element of the range, it is said to be many-to-one. If each element of the range is mapped onto a single element of the domain, the function is said to be one-to-one.

1) The following function maps an element x onto its image f(x) = y.

 $f: x \to 2x - 3.$ 

- (a) Find the range of the function for its domain  $\{0 \le x \le 4\}$ .
- (b) Explain why f is a one-to-one function.
- 2) (a) What is the name of the graph of a quadratic function? What shape is that graph? Does such a graph have a line of symmetry?
  - (b) Function  $x \to x^2$  has the domain  $\mathbb{R}$  because it is defined for all real x. (i) Explain why the range will contain no negative number.

Therefore, the range is  $\{y \in \mathbb{R} : y \ge 0\}$ 

(ii) Give an example to show that some elements of the range can be obtained from more than one element of the domain.

(iii) Is the function many-to-one or one-to-one?

3) (a) Construct a table of values for the function f : x → 2x - 3. How many values do you need to plot the graph of f? Explain how you would draw it.
 (b) State whether the point (12.5, 21) lies on the graph

(b) State whether the point (12.5, 21) lies on the graph.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

#### MAPPING

# Sujet n° D1\_82

The first part of this page is a summary that can help you do the exercise.

Linear function:
A linear function is in the form f(x) = mx + p.
If two points A(x<sub>A</sub>, y<sub>A</sub>) and B(x<sub>B</sub>, y<sub>B</sub>) - with x<sub>A</sub> ≠ x<sub>B</sub> - belong to the graph of a linear function, the gradient *m* is given by the formula <sup>y<sub>A</sub>-y<sub>B</sub></sup>/<sub>x<sub>A</sub>-x<sub>B</sub></sub>.
Quadratic function:
A quadratic function is in the form f(x) = ax<sup>2</sup> + bx + c with a ≠ 0.
The *x*-coordinate of the vertex of its graph is equal to -<sup>b</sup>/<sub>2a</sub>.
To solve ax<sup>2</sup> + bx + c = 0 with a ≠ 0, you can use the quadratic formula <sup>-b±√b<sup>2</sup>-4ac</sup>/<sub>2a</sub>.

Two brothers live in the same house and their bedrooms are on two different floors. The younger brother's room is above the other one.

At the same time, they throw something in their garden from the window of their own room: the younger throws a stone and the elder a tennis ball.

We want to study the paths of their projectiles in terms of the time.

Let's label t the time, S(t) the path followed by the stone and B(t) the path followed by the tennis ball.

#### 1) Path of the stone:

At t = 0 s, the younger child throws a stone which follows a straight line. At t = 1 s, the height of the stone is 4 m and at t = 3 s, the stone is 2m above the ground.

- a) Find the expression of S(t) in terms of t.
- b) What is the height of the stone at the beginning of the throw?
- c) When does the stone reach the ground?

# 2) Path of the tennis ball:

The tennis ball follows a quadratic curve.

At the beginning of the throw, the ball is 1 m high. 2s after the beginning, the ball is 5 m high and at t = 3s, the ball is 4 m above the ground.

a) Prove that the path of the tennis ball is given by the expression

 $B(t) = -t^2 + 4t + 1.$ 

- b) Describe the path followed by the tennis ball. When does the ball reach its maximum height?
- c) When does the ball reach the ground?
- 3) When are the two projectiles at the same height?
#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## MAPPING Sujet n° D1\_83

The first part of this page is a summary that can help you do the exercise.

A quadratic function is in the form  $f(x) = ax^2 + bx + c$  with  $a \neq 0$ . The *x*-coordinate of the vertex of its graph is equal to  $-\frac{b}{2a}$ . To solve  $ax^2 + bx + c = 0$  with  $a \neq 0$ , you can use the quadratic formula  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

Alfred's rectangular garden is composed of a lawn and a path. His garden is 30 by 16 yards. He wants to make a path all around the lawn such that the area of the path is equal to half the area of the garden. The width of the path must be the same everywhere.



1) What is the area of the garden?

- 2) Let x be the width of the path. Show that the area of the path is  $-4x^2 + 92x$ .
- 3) Show that an equation to find the width of the path is  $x^2 23x + 60 = 0$ .

4) Solve the equation and conclude the problem.

5) Later, he changed his mind and he wants to make two perpendicular paths of the same width like below:



We still want the area of the path to be half the area of the garden. What must the width of the path be?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## MAPPING Sujet n° D1\_84

The first part of this page is a summary that can help you do the exercise.

The standard form equation of a quadratic function is:  $f(x) = ax^2 + bx + c$  with *a*, *b* and *c* constants,  $a \neq 0$ . The graph of *f* is called a parabola ; the abscissa of its vertex is  $-\frac{b}{2a}$ .

Two companies A and B produce and sell 500 g corn flakes packs for £5 each.

1) (a) The cost of production in pounds, for company *A*, is given for *q* units produced and sold by:

$$C(q) = 0.01q^2 - 10q + 2510.$$

How many units should be produced and sold so that the cost is minimum?

- (b) Let B(q) be the profit in pounds for q units produced and sold. Explain why  $B(q) = -0.01q^2 + 15q - 2510$ .
- (c) Do you think that the profit is maximum when the cost of production is minimum?
- 2) The two companies decide to lower their prices: company *A* offers 10% off the price for each pack, while company *B* offers 10% more product for the same price of £5. Show that company *A* gives the better offer.
- 3) On each pack of corn flakes, company A uses a friendly logo made out of two intersecting parabolas, as shown below. Find the expression of f(x) in terms of x?



ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# DIFFERENTIATION

D2 - 71

**Tools** Given a right triangle ABC in B

- The Pythagoras theorem states  $AB^2 + BC^2 = AC^2$
- The surface of the triangle is given by  $\frac{AB \times BC}{2}$

## **Product Rule**

Given two derivable functions u(x) and v(x) we have  $\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$ 

An artist has been commissioned to make a stained glass window in the shape of an octagon. He wants to minimize the surface of the glass he will use in order to minimize the costs.

The octagon must fit inside an 4-in. square space. See the figure below :



- 1. The hypotenuse of each triangle must be 2 in. long. Let x be the length of one leg. Explain how to find and compute the expression of the length of the other leg.
- 2.
- a. Show the area of one triangle must be  $A(x) = \frac{x\sqrt{4-x^2}}{2}$ .
- b. What is the domain of A(x)?
- c. Compute the derivative  $\frac{dA}{dx}$ .
- 3. What is the minimum surface area of glass?

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

## DIFFERENTIATION

D2 - 72

**Tools** Surface of a circle, with a radius  $r : \pi r^2$ Perimeter of a circle with a radius  $r : 2\pi r$ 

Liam wants to order cylindrical cups for his birthday party from a cup factory.

The external and base surface area must be  $48\pi$  cm<sup>2</sup>.

This diagram shows a cylindrical container of radius r cm and height h cm. The container has an open top and a circular base.



When the radius of the base is r cm, the volume of the container is V cm<sup>3</sup>. Liam wants to find if there are values for h and r that maximize the volume of the cup and what they are if they exist.

1) Show that  $h = \frac{48-r^2}{2r}$  and deduce that  $V = 24\pi r - \frac{\pi}{2}r^3$ .

2)

a. Find the derivative  $\frac{dV}{dr}$ .

b. Find the values of r for which V has a stationary point.

3) In order to maximize the volume find the dimension of the cups that Liam must ask for.

After AQA-General Certificate of Education Advanced Subsidiary Examination, June 2015.

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

#### Differentiation

D2 - 81

#### Tools

- The volume of a rectangular parallelepiped is  $V = a \times b \times c$  where *a* is the width, b the length and c the height (all expressed in the same unit).

- The total surface area is equal to the sum of the surface areas of all the faces.

The diagram shows a block with a base measuring 2x cm by x cm and a height of h cm. The total surface area of the block is  $300 \text{ cm}^2$ .



1) Show that the total surface area of this block is:  $S = 4x^2 + 6hx$ .

2) Show that 
$$h = \frac{150 - 2x^2}{3x}$$

- 3) Express the volume of the block in terms of x.
- 4) Find the value of *x* that gives the block a maximum volume and calculate this maximum volume.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

#### DIFFERENTIATION

D2 - 82

### **Designing a Suitcase**

A 24- by 36-in. sheet of cardboard is folded in half to form a 24- by 18-in. rectangle as shown in the figure.

Then four congruent squares of side length x are cut from the corners of the folded rectangle. The sheet is unfolded, and the six tabs are folded up to form a box with sides and a lid.



The sheet is then unfolded.



- 1. Write down a formula for the volume V(x) of the box.
- 2. Check that  $V(x) = 8x(x^2 21x + 108)$ .
- 3. Find the domain of V for the problem situation.
- 4. Using your calculator, plot *V* over its domain. (hint  $X_{min} = 0$ ,  $X_{max} = 10$ ,  $Y_{min} = 0$ and  $Y_{max} = 1400$ )
- 5. Use the graph to find an approximate value of the maximum volume and the value of *x* that gives it.
- 6. Confirm the result you found in the previous question by a calculation. (hint: differentiate V)
- 7. Find a value of x that yields a volume of  $1100 \text{ in}^3$ .

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## SEQUENCES

Sujet D3 - 52

The first part of this page is a summary that can help you do the exercise.

In a geometric sequence, each term can be obtained by multiplying the previous term by a constant value. This value is called the common ratio, or r.

You can find the *n*th term of a geometric sequence using the formula  $u_n = u_1 \times r^{n-1}$ .

On January the 1<sup>st</sup>, Harry invests £2000 in an account which pays 5% interest per month.

At the start of each month, starting from February, he spends £200. The interests are paid on the 20<sup>th</sup> of each month.

Let  $u_n$  be the amount in his account **at the end** of the *n*th month (January is the first month).

- **1.** Find the values of  $u_1$  and prove that  $u_2 = 1995$ .
- **2.** Express  $u_{n+1}$  in terms of  $u_n$ .
- **3.**  $(V_n)$  is the sequence defined by  $V_n = u_n 4200$ .
  - **a.** Prove that  $(V_n)$  is a geometric sequence and find its common ratio and the value of  $V_1$ .
  - **b.** Express the *n*th term  $V_n$  in terms of *n* and then the *n*th term of  $u_n$  in terms of *n*.
- 4. After how many months will he be in debt?
- 5. Does such an interest rate seem realistic?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## SEQUENCES

Sujet D3 - 62

The first part of this page is a summary that can be useful to do the exercise.

## Sequences

A geometric progression (G.P.) is a sequence in which each term after the first is formed by multiplying the previous term by a fixed number, called the common ratio. If  $a_1$  is the first term and r is the common ratio, the *n*th term is:  $a_n = a_1 \times r^{n-1}$ .

An arithmetic progression (A.P.) is a sequence in which each term after the first is formed by adding a fixed number to the previous term, called the common difference. If  $a_1$  is the first term and r is the common difference, the *n*th term is:  $a_n = a_1 + (n-1)r$ .

## Exercise:

Thomas Malthus (1766 – 1834) was an English cleric and scholar, influential in the fields of political economy and demography. In his Essay on the Principle of Population (first edited in 1798), based on statistical figures of the 18<sup>th</sup> century, he argued that population multiplies geometrically and food arithmetically; therefore, the population will eventually outstrip the food supply.

In 1801, the population in England was of 8 million inhabitants and the rate of growth of the population was 2% per year. Meanwhile, the British agriculture was able to feed 10 million inhabitants, and Malthus estimates each year, it could feed an extra 0.4 million inhabitants.

For  $n \ge 1$ , let  $P_n$  be the population in year 1800 + n, and let  $F_n$  be the population that can be fed in year 1800 + n.

- 1. Explain why  $P_n$  is a geometric sequence. Give the common ratio, the first term and expression of the *n*th term.
- 2. Explain why  $F_n$  is an arithmetic sequence. Give the common difference, the first term and expression of the *n*th term.
- 3. According to Malthus, what will be the population in England in 1850? How many inhabitants can be fed in England in 1850?
- 4. Use any method of your choice to find out when it's not possible to feed the whole population anymore.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## SEQUENCES

Sujet D3 - 63

#### Summary :

#### Arithmetic sequence :

A sequence  $\{a_n\}$  is an arithmetic sequence (or arithmetic progression) if it can be written in the form :  $a_n = a_{n-1} + d$ , where  $n \ge 2$ , for some constant *d*.

The number *d* is the common difference.

Let  $a_1$  be the first term.

The *n*th term of an arithmetic sequence can be written in the form  $a_n = a_1 + (n-1)d$ 

#### Arithmetic series :

A series is an arithmetic series if its terms form an arithmetic sequence.

The sum  $S_n$  of the arithmetic series  $a_1 + a_2 + a_3 + ... + a_k + ... + a_n$  with common difference d is

$$S_n = n \left( \frac{a_1 + a_n}{2} \right)$$
 or  $S_n = \frac{n}{2} \left[ 2a_1 + (n-1)d \right]$ 

#### Exercise:

We draw a succession of interlocked pentagons P<sub>1</sub>, P<sub>2</sub>, P<sub>3</sub>, P<sub>4</sub> as shown below:



 $u_n$  is the number of dots in each figure, it is called the  $n^{\text{th}}$  pentagonal number.

 $u_1 = 1$ 

 $u_2 = 5$ 

 $u_3 = 12$ 

1. Give the value of  $u_4$ .

2.

- a. How many dots are there on one side of the  $n^{th}$  pentagon  $P_n$ ?
- b. Prove that  $u_{n+1} = u_n + 3n + 1$ . If you cannot prove it, move on to the next question.

3. For  $n, n \ge 1$ ,  $v_n = u_{n+1} - u_n$ .

a. Prove that  $(v_n)$  is an arithmetic sequence (A.P).

b. Give  $v_1 + v_2 + v_3 + ... + v_{n-1}$  in terms of *n*.

- c. Write  $v_1 + v_2 + v_3 + \ldots + v_{n-1}$  in terms of  $u_n$ . Then, give the expression of  $u_n$  in terms of n.
- 4. Give the 10<sup>th</sup> pentagonal number.

## ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## SEQUENCES

### Sujet D3 - 65

The first part of this page is a summary that can help you do the exercise.

#### Arithmetic sequence

A sequence  $(a_n)$  is an arithmetic sequence with common difference d if it can be written  $a_{n+1} = a_n + d$ , where  $n \ge 1$ .

The *n*-th term of an arithmetic sequence whose first term is  $a_1$  and common difference *d* can be written  $a_n = a_1 + (n-1)d$ .

The sum of the first *n* terms of an arithmetic sequence is:  $S_n = \frac{n}{2}(a_1 + a_n)$ .

#### Geometric sequence

A sequence  $(a_n)$  is a geometric sequence with common ratio r if it can be written  $a_{n+1} = r. a_n$ , where  $n \ge 1$ . The *n*-th term of a geometric sequence whose first term is  $a_1$  and common ratio r can be written  $a_n = a_1 r^{n-1}$ .

When r \neq 1 the sum of the first n terms of a geometric sequence is  $S_n = a_1 \times \frac{1-r^n}{1-r}$ .

A well is a deep hole made in the ground through which water can be removed.

#### EXERCISE

A company wants to drill a deep well into the ground, and contacts two businesses A and B.

With business A, the first metre to be drilled costs £50, and each extra metre costs £10 more than the previous one.

With business B, the first metre to be drilled costs  $\pounds40$ , and each extra metre costs 10% more than the previous one.

#### Business A

**1.** Let  $a_n$  be the price of the  $n^{\text{th}}$  metre to be drilled into the ground by business A.

(a) Find  $a_1$ ,  $a_2$ , and  $a_3$ ; what can you say about the sequence ( $a_n$ )?

(b) Write  $a_n$  in terms of n.

**2.** Let  $A_n$  be the price of a drilling of total length n metres with business A.

(a) Find  $A_1$ ,  $A_2$ , and  $A_3$ ; what can you say about the sequence  $A_n$ ?

(b) Explain why  $A_n = n(5n+45)$ .

#### **Business B**

**3.** Let  $b_n$  be the price of the *n*th metre to be drilled by business B, and  $B_n$  the price of a *n*-metre drilling with business B. Find  $b_n$  in terms of *n*. Show that  $B_n = 400(1.1^n - 1)$ 

#### Business A or business B?

**4.** The company has to drill a 43-metre-long well into the ground: could you figure out which offer seems to be the best and what the final cost would be?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

# SEQUENCES

D3 - 71

A sequence in which the differences between successive terms are equal is called an arithmetic sequence or arithmetic progression.

The number added to each term to get the next one is called the common difference.

If the initial term of an arithmetic progression is  $a_1$  and the common difference is d, then the  $n^{th}$  term of the sequence, called the general term, is given by:  $a_n = a_1 + (n-1)d$ .

 $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{n}$ 

Jeremy, a bee-keeper, is doing an investigation which involves building hexagon-shaped patterns with sticks. His first three patterns are shown below.



1) What is the extra number of hexagons and the extra number of sticks necessary in step 4?

Step n	1	2	3	4			
Number of	1	2					
hexagons on row $n$ : $h_n$		2					
Total number of	1						
hexagons at step $n$ : $H_n$							
Extra number of	6	0					
sticks on row $n$ : $T_n$	0	9					
Total number of	6						
sticks used at step $n$ : $S_n$	U						

2) Copy and complete the table below:

1) **Number of hexagons**: express the total number of hexagons at step n in terms of n. Explain your answer.

#### Number of sticks:

2)

- a) Using the table above, express  $T_{n+1}$  in terms of  $T_n$ . What type of sequence is  $T_n$ ?
- b) Express  $T_n$  in terms of n.
- 3) Express  $S_{n+1}$  in terms of  $S_n$  and n.
- 4) Jeremy would like to find a formula to express  $S_n$  in terms of n. Here are three formulae. Help him choose the right one. (Explain your choice)  $S_n = 2n^2 + 4n$   $S_n = \frac{3}{2}n^2 + \frac{9}{2}n$   $S_n = \frac{5}{2}n^2 + \frac{3}{2}n + 2$

5) Jeremy used 105 sticks to build his last pattern. How many rows are there at this step? How many hexagons are there?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

# SEQUENCES

D3 - 72

#### Arithmetic sequences

A sequence  $(a_n)$  is an arithmetic sequence if it can be written in the form  $a_n = a_{n-1} + d$  where  $n \ge 1$ . Number *d* is called the common difference. The *n*th term of an arithmetic sequence can be written in the form  $a_n = a_1 + (n-1)d$ , where  $a_1$  is

the first term and d is the common difference.  $a_1 + a_2$ 

The sum  $S_n$  of an arithmetic series  $a_1 + a_2 + a_3 + \dots + a_k + \dots + a_n$  with  $d \neq 1$  is  $S_n = n \times \frac{a_1 + a_n}{2}$ .

In your new 'get-fit' program, you plan to jog 1,500 metres on the first night and then increase this distance by 250 metres each subsequent night.

1. First, focus on the distance jogged each night, and complete the table.

Night number	1	2	3	4
Distance jogged (m)				

- 2. If you continue the pattern, write down an expression for  $D_n$ , the distance jogged on the *n*th night.
- 3. How far will you jog :
- a) on the 7th night ?
- b) on the 12th night?
- 4. Now focus on the total distance jogged over several nights, and complete the table.

Night number	1	2	3	4
Total distance jogged (m)				

- 5. Write down an expression for  $S_n$ , the total distance jogged after *n* nights.
- 6. Determine the total distance you expect to jog after ten nights.
- 7. Determine the number of nights you will need to ensure that you jog a total of more than 50km.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

## SEQUENCES

#### D3 - 81

#### Arithmetic and geometric sequences

The *n*th term of an arithmetic sequence can be written in the form  $a_n = a_1 + (n - 1)d$ , where  $a_1$  is the first term and *d* is the common difference.

The *n*th term of an geometric sequence can be written in the form  $b_n = b_1 * q^{n-1}$ , where  $b_1$  is the first term and *q* is the common difference.

The sum $S_n$ of the arithmetic series $a_1 + a_2 + a_3 + \dots + a_k + \dots + a_n$ with $d \neq 1$ is $S_n = n \times \frac{a_1 + a_n}{2}$	•
The sum of the geometric series $b_1 + b_2 + + b_n = b_1 \times \frac{1-q^n}{1-q}$	

## The third labour of Heracles: the capture of the Ceryneian Hind

The Greek hero Heracles (or Hercules) had to carry out 12 extraordinary labours given by King Eurystheus in order to explate one terrible crime.

The third labour was to capture the Ceryneian Hind, a hind so fast it could outrun an arrow. As Heracles finds the hind, the startled animal starts to run and Heracles has no choice but to chase it.

- The first day, the Ceryneian Hind runs 50 kilometers and since it is chased, each day it will run 10 more kilometers than the day before.
- The first day, Heracles will run 40 kilometers and since he really needs to catch the hind, each day he will run a 5% longer distance than the day before.

Let  $u_n$  denote the distance run by the hind on the n-th day and  $v_n$  denote the distance run by Heracles on the n-th day. So  $u_1$ =50 and  $v_1$ =40

- 1. Calculate the distance run by the Ceryneian Hind and Heracles the second day.
- 2. Calculate the distance to the nearest kilometer run by the Ceryneian Hind and Heracles the tenth day.
- 3. Find out smallest index n for which  $u_n < v_n$ . What does that mean?
- 4. Using your calculator, find out how many days Heracles needs to catch up with the Ceryneian Hind. Calculate the total distance to the nearest kilometer run by the animal and the hero before he manages to capture it.



ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

# D3 - 82

The first part of this page is a summary that can be useful to do the exercise.

## Sequences

A geometric progression (G.P.) is a sequence in which each term is obtained by multiplying the preceding term by a fixed number, called the common ratio. If  $a_1$  is the first term and r is the common ratio, the *n*th term is:  $a_n = a_1 r^{n-1}$ .

An arithmetic progression (A.P.) is a sequence in which each term is obtained by adding a fixed number, called the common difference, to the preceding term. If  $a_1$  is the first term and r is the common difference, the *n*th term is:  $a_n = a_1 + (n-1)r$ .

## Exercise

On a tweeting social network, Albert realizes that every morning, he gets 10% more followers than the day before, but then 5 followers change their minds and unfollow him during the rest of the day.

Let  $(a_n)$  be the number of followers Albert has at the end of the *n*th day.

- 1. Explain why  $a_n = 1.1a_{n-1} 5$ , for every integer *n*.
- 2. If Albert has 50 followers at the end of the first day, how many friends will he have by the end of the month (30 days later)? By the end of the year (365 days later)?
- 3. What happens if Albert has 50 friends at the end of the first day?
- 4. What happens if Albert has less than 50 friends at the end of the first day?
- 5. We assume that Albert has 51 friends at the end of the first day.
  - a. Let  $(u_n)$  be a sequence such that  $u_n = a_n 50$  for every integer n.

Explain why  $(u_n)$  is a geometric progression.

- b. Find the expression (only in terms of *n* and  $u_1$ ) of the general term of  $(u_n)$ . Prove that  $a_n = 1.1^n + 50$ .
- c. There are 300 million active members on the tweeting social network. When will Albert be theoretically followed by all of the active members?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

#### SEQUENCES

#### D3 - 83

The first part of this page is a summary that can be useful to do the exercise.

### Sequences

A geometric progression (G.P.) is a sequence in which each term is obtained by multiplying the preceding term by a fixed number, called the common ratio. If  $a_1$  is the first term and r is the common ratio, the *n*th term is:  $a_n = a_1 r^{n-1}$ .

An arithmetic progression (A.P.) is a sequence in which each term is obtained by adding a fixed number, called the common difference, to the preceding term. If  $a_1$  is the first term and r is the common difference, the *n*th term is:  $a_n = a_1 + (n-1)r$ .

## Exercise

One fine day, Mary and John chose two different ways of saving money.

- John first put £500 into a box and decided to add £150 each month.
- Mary first put £5 into her own box, deciding to double its content every month.

Let us call  $j_n$  and  $m_n$  the total amount of money John and Mary have respectively saved after n months.

- 1. What are the values of  $j_1$  and  $m_1$ ?
- 2. How much money will each of them have at the beginning of the 2<sup>nd</sup> month? At the beginning of the 3<sup>rd</sup> month?
- 3. Express  $j_n$  in term of n and  $j_1$ .
- 4. Express m<sub>n</sub> in term of n and m<sub>1</sub>.
- 5. Prove that John will have saved £2,300 and Mary £20,480 after one year,
- 6. Use your calculator to find the first month when Mary will become richer than John.

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

### Binôme : Anglais / Mathématiques

# D3 - 84

The first part of this page is a summary that can be helpful to do the exercise.

## Arithmetic and geometric sequences

An arithmetic progression (A.P.) is a sequence in which each term after the first is formed by adding a fixed amount, called the common difference, to the preceding term. If  $a_1$  is the first term and d is the common difference, the *n*th term is:  $a_n = a_1 + (n-1)d$ 

A geometric progression (G.P.) is a sequence in which each term after the first is formed by multiplying the preceding term by a fixed number, called the common ratio. If  $b_1$  is the first term and *r* is the common ratio, the *n*th term is:  $b_n = b_1 r^{n-1}$ .

The sum of two sequences  $(u_n)$  and  $(v_n)$  is the sequence  $(w_n)$  defined for all integer *n* by

 $w_n = u_n + v_n$ .

- **1.** Prove that the sequence  $u_n = n^2 1$  is neither an AP nor a GP.
- 2. Is the sum of two APs always an AP?
- 3.
- a) Let  $u_n = 3 \times 2^n$  and  $v_n = 3 \times 4^n$ . Is  $u_n + v_n$  a GP?
- b) Let  $u_n = 2 \times 3^n$  and  $v_n = 5 \times 3^n$ . Is  $u_n + v_n$  a GP?
- c) Discuss in which cases the sum of two given GPs is a GP.
- **4.** Let  $(a_n)$  be the sequence defined by  $a_1 = 1$  and  $a_{n+1} = 3a_n 4$  for all integer *n*.

Is the sequence  $b_n = a_n - 2$  a GP?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

### Binôme : Anglais / Mathématiques STATISTICS

## Sujet D4 – 41

#### The first part of this page is a summary which may help you solve the following exercise.

**Scatter graphs** are used to compare two sets of data. One set of data is put on the x-axis and the other on the y-axis. Usually, to see if there is a relationship between the two sets of data, we can draw a line of best fit. If the points are close to the best-fit line, we say that there is a **strong correlation**. If the points are loosely scattered, there is a **weak correlation**. Also, if the best fit line slopes upwards, there is a **positive correlation**. If the line slopes down, there is a **negative correlation**.

#### Exercise:

- 1) There are two routes for a worker to get to his office. Both routes involve delays due to traffic lights. He records the time it takes over a series of several journeys for each route.
  - The results are shown below.

	mean	standard deviation
Route 1	22 min	10 min
Route 2	25 min	5 min

Suggest which route you would recommend. State your reason clearly.

2) The scatter graph shows the age of cars and the number of kilometers travelled.



- (a) Describe the relationship shown by this scatter graph.
- (b) The age and number of kilometers travelled by one of these cars look out of place.
  - (i) What is the age of this car and how many kilometers has it travelled?

(ii) Give a possible reason why the results for this car are different from the rest of the cars.

(c) We extract the former car from the set of data.

(i) Use your calculator to find an equation of the line of best fit, giving the number y of kilometers travelled in terms of the age x of the car. Give your result to the nearest unit.

- (ii) What could be the age of a car that has travelled 35000km? Give your result to 1dp.
- (iii) What could be the mileage of a 6 year old car? Of a 10 year old car?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## **STATISTICS**

#### Sujet D4 - 42

The first part is a summary that can help you do the exercise.

The **arithmetic mean** of the values  $x_1, x_2, ..., x_p$  whose corresponding frequencies are  $n_1, n_2, ..., n_p$ , is

$$\overline{x} = \frac{n_1 x_1 + n_2 x_2 + \dots + n_p x_p}{n_1 + n_2 + \dots + n_p}.$$

**Scatter graphs** are used to compare two sets of data. One set of data is put on the x-axis and the other on the y-axis. Usually, to see if there is a relationship between the two sets of data, we can draw a line of best fit. If the points are close to the best-fit line, we say that there is a **strong correlation**. If the points are loosely scattered, there is a **weak correlation**. Also, if the best fit line slopes upwards, there is a **positive correlation**. If the line slopes down, there is a **negative correlation**.

The table alongside shows the *percentage of unemployed adults* and the *number of major thefts per day* in eight large cities.

a) Find the mean percentage x and the mean number of thefts  $\overline{y}$ .

b) Draw a scatter diagram for this data.

c) Describe the correlation between the *percentage of unemployed adults* and the *number of major thefts per day*.

d) Plot  $(\overline{x}, \overline{y})$  on the scatter diagram.

e) Using your calculator, find an equation of the line of best fit, then draw it on the scatter diagram.

f) Another city has 8% unemployment. Estimate the number of major thefts per day for that city.

g) What could be the percentage of unemployment for a city that has 40 major thefts per day?

Source : IGCSE Cambridge International Mathematics (0607) Extended

City	Percentage	Number
А	7	113
В	6	67
С	10	117
D	7	82
Е	9	120
F	6	32
G	3	61
Н	7	76

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

STATISTICS

Sujet D4 - 51

#### Stem & leaf and Histogram

The first part of this page is a summary that can help you do the exercise.

Reminder :
Let $(x_1, x_2, \dots, x_N)$ be a distribution of N numbers.
The <b>mean</b> is given by $\frac{x_1 + x_2 + \dots + x_N}{N}$ .
If <i>N</i> is even, then the <b>median</b> is the mean of the two middle values. If <i>N</i> is odd, then the <b>median</b> is the middle value.
The <b>first quartile</b> $Q_1$ is given by $x_i$ where <i>I</i> is $\frac{N}{4}$ , rounded to the next integer.
The <b>third quartile</b> $Q_3$ is given by $x_J$ where J is $\frac{3N}{4}$ , rounded to the next integer.
The <b>modal value</b> is the most common value.
A histogram displays frequencies from a grouped frequency distribution: the frequency of a class
interval is represented by a rectangle whose surface area is proportional to the frequency.
A group of 20 students took a math test. The school board had the tests graded by two applicant teachers (Ap1 and Ap2). The grades are shown in the back-to-back stem-and-leaf diagram below. The board of directors would like to hire one math teacher. Here are the marks :

Ap1	1	1	Ap2
		2	
		3	9
		4	4 5 7
	98730	5	3 4 4
	987763	6	013679
	5432111	7	2235
	0	8	0 5
		9	3
	key: 7   4   means 47		key:  8 0 means 80

inspired from "Concise Maths 4&5 – for Leaving Cert higher level – Project maths supplement" by George Humphrey – Brendan Guildea – Geoffrey Reeves – Louise Boylan Gill & MacMillan editor 2010

#### **Questions**

- 1. Did the first applicant give the mark 68? And did Ap2 give the mark 46?
- **2.** Compute the mean, median, interquartile range for both distributions.
- 3. Find the modal values. Explain how to retrieve them from the stem-and-leaf plot.
- **4.** From the measurements that you have computed, would you rather hire Ap1 or Ap2? Explain the reasons for your choice.
- **5.** Draw a histogram for the grades given by Ap2. Describe the relationship between the stem-and-leaf plot and the histogram.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## STATISTICS

#### Sujet D4 - 52

The first part of this page is a summary that can help you to do the exercises.

The most commonly used averages for a set of data are the mode, the median and the mean. The mode is the most common value in a set of data.

In a list of numbers in order of size, the median is the middle number.

If there are *n* numbers in the list, with *n* being an even number, the median is half the sum of the  $(\frac{1}{2}n)^{\text{th}}$  number and the  $(\frac{1}{2}n+1)^{\text{th}}$  number.

The mean of a set of numbers is the sum of the numbers divided by the quantity of numbers.

The correlation coefficient measures the strength of the linear relationship between two sets of data.

If the points on a scatter graph lie close to a straight line, a strong correlation is said to exist and the correlation coefficient is near 1 (with a positive correlation) or -1 (with a negative correlation).

#### Exercise 1

The annual salaries of the employees in a small company are listed below in ascending order: € 20,000 €22,000 €24,000 €25,000 €30,000 €105,000

The mean salary is €37,667 rounded to the nearest whole number.

- **1.** Find the median salary.
- 2. Why can't you find the mode?
- 3. Which of the averages, mean or median, best represents the 'typical' salary?

#### Introduction to exercise 2

Reading is fundamental in today's society. There are many adults who cannot read well enough to understand the instructions on a medicine bottle. This is a scary thought - especially for their children. Filling out applications becomes impossible without help. Reading road or warning signs is difficult. Even following a map becomes a chore. Day-to-day activities that many people take for granted become a source of frustration, anger and fear.

It is important to realize that struggling with vital reading skills is not a sign of low intelligence. Many highly intelligent people have struggled with reading, although, when properly taught, most people can learn to read easily and quickly.

## Exercise 2

This scatter graph shows the number of books read by some children and the ages of these children.



- 1. How many children have read more than 100 books?
- 2. One of these children has read 50 books. How old is this child?
- 3. Describe the relationship shown by the scatter graph.

(The correlation coefficient is r = 0.92).

**4.** The equation of the line of regression, or line of best fit, is y = 0.02 x + 7.7. A new child joins the group, he has read 170 books. Assuming this child shares many characteristics with the other children, estimate his age.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

## STATISTICS

#### Sujet D4 - 53

The first part of this page is a summary that can help you do the exercise.

Let's consider a set of numerical values.

• The **mode** is the most common value.

• The **mean** is the sum of all values divided by the total number of values.

• The **median** is the middle value when the values are listed in order. If there are two middle values (because there is an even number of values) the median is the mean of these two middle values.

The range is the difference between the highest value and the lowest value.

EXERCISE Questions 1 and 2 are independent, and can be answered separately.

#### Question 1 The best route

James has two possible ways of getting to school:

A: He can catch a bus to the town centre and another bus out; or

**B**: He can walk in a different direction and then just catch another bus.

He isn't sure which way is quicker so he decides to time his journey from leaving his front door to arriving at the school gate. He times 20 journeys by route *A* and 20 by route *B*. Here are his results (in minutes).

Route <b>A</b>	24	18	14	19	22	23	18	15	22	30
	30	26	22	27	16	22	26	19	19	27
Route <b>B</b>	17	17	18	18	18	19	20	20	21	21
	21	22	24	24	24	24	25	25	26	28

a) The mean, median, mode and range for the times of route *A* have already been calculated (see table below). Work out these measures of average and spread for route *B*.

route	mean	median	mode	range
A	21.95	22	22	16

**b)** Which route do you think James should use, and why?

#### Question 2 Possible or Impossible?

I have a set of numerical values which could be anything, and I want to know whether each of these statements (separately) is possible or impossible.

Can it apply to the mean, the median, the mode or the range?

If it's possible, give an example; if impossible, explain why (some answers are given to help you).

Statement	Mean	Median	Mode	Range
It's equal to zero.		Possible; e.g. -10, -1, 0, 2, 3		
It's the highest value.				
It's less than any of the values.			Impossible, because the mode must be an actual value.	

## ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

## Binôme : Anglais / Mathématiques

## STATISTICS

#### Sujet D4 – 54

The first part of this page is a summary that can help you to do the exercises.

<u>Recap</u>: 1 inch = 2.54 cm; 1 pound = 0.454 kg

The **mean** of a set of numbers is the sum of the numbers divided by the quantity of numbers.

The **correlation coefficient** measures the strength of the linear relationship between two sets of data.

If the points on a scatter graph lie close to a straight line, a **strong correlation** is said to exist and the correlation coefficient is near 1 (with a positive correlation) or -1 (with a negative correlation).

You may use your calculator to find the correlation coefficient and the least squares regression line in statistics.

#### Exercise:

The heights and weights of a group of 9 pupils are recorded in the following table:

Height (inches)	64	68	65	55	75	58	72	60	63
Weight (pounds)	134	142	137	124	150	128	147	129	132

- 1) Calculate the mean height of the group and give your answer in m.
- 2) Calculate the mean weight of the group and give your answer in kg.
- 3) Explain how to draw a scatter graph for this data and draw it as best as you can.Is it appropriate to fit a straight line for this data?
- 4) How strong is the correlation between the height and the weight?
- 5) Draw the line of best fit on your scatter graph.
- 6) Predict the weight of a pupil who would be:
  - (i) 66 inches high (ii) 39 inches high
- 7) Why might the previous answer be unreliable?

## ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

## Binôme : Anglais / Mathématiques

## STATISTICS

#### Sujet D4 – 55

The first part of this page is a summary that can help you to do the exercises.

In a list of numbers in order of size, the median is the middle number.

If there are *n* numbers in the list, with *n* being an even number, the **median** is half the sum of the  $(\frac{1}{2}n)^{\text{th}}$  number and the  $(\frac{1}{2}n+1)^{\text{th}}$  number.

The mean of a set of numbers is the sum of the numbers divided by the quantity of numbers.

The **correlation coefficient** measures the strength of the linear relationship between two sets of data.

If the points on a scatter graph lie close to a straight line, a **strong correlation** is said to exist and the correlation coefficient is near 1 (with a positive correlation) or -1 (with a negative correlation).

#### Exercise:

**1)** The times spent by a class of 30 pupils revising a Math test has been collected in the following table.

Time spent (in minutes)	Less than 10	10-20	20-30	30-40	40-50	50-60
Number of pupils	2	4	8	6	7	3
Cumulative frequency	2					

- **a.** Fill in the third row of the table above and draw a cumulative frequency curve to estimate the median time, the lower quartile and the upper quartile. Comment your answers.
- **b.** Find the interquartile range. Comment your answer.
- 2) We want to know if there is a relationship between the number of minutes spent revising the test and the number of mistakes that are made. A random sample of 9 pupils is listed below.

Time spent (in minutes)	8	18	24	27	33	35	37	41	54
Number of mistakes	12	9	7	7	6	12	4	3	1

- **a.** Plot the corresponding points in a scatter graph.
- **b.** What type of correlation is shown?
- **c.** Draw a line of best fit on your scatter graph.
- d. A pupil revised for 25 minutes. Estimate the number of mistakes he made.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

## Binôme : Anglais / Mathématiques

## STATISTICS

Sujet D4 - 61

#### Part A

A math teacher, Mr. Mat, registered the results of his students in the following table :

	· · · · · · · · · · · · · · · · · · ·			5
Mark x	$0 \le x < 5$	$5 \le x < 10$	$10 \le x < 15$	$15 \le x \le 20$
Frequency	4	8	12	6

- 1. What type of data is this?
- 2. Explain how you can find the median of this data.
- **3.** An estimate of the median is 11. Explain what it means.
- 4. Prove that the interquartile range is roughly equal to 7.

#### Part B

Mr. Mat wants to compare the results of his students with the marks of his colleague's class (Mrs Smith's) for the same test. The median she found is 13, and the interquartile range she had is 4. Compare the results of the 2 classes.

#### Part C

Mr. Mat also wants to know if there is a link between the math results of 5 of his students and their English results. This is the table which shows the means of these students in math and English:

Math mean x	12	14	9	11	16
English mean y	11	16	9	12	17

**1.** This scatter graph illustrates these results:



Explain the correlation between the math and the English results.

- **2.** A student has got a math mean of 13. Use the line of best fit to predict his English mean.
- 3. Another student has got an English mean of 2. Could you predict his math mean?

#### BACCALAURÉATS GÉNÉRAL ET TECHNOLOGIQUE SESSION 2018 ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## STATISTICS

Sujet D4 – 71

The first part of this page is a summary that can help you to do the exercises.

The frequency density is equal to the frequency of a class interval divided by its width. For continuous data, you obtain the median thanks to a cumulative frequency curve: it is the xcoordinate of the point whose y-coordinate is half of the total frequency. You can use your calculator to find the correlation coefficient and the least squares regression line

You can use your calculator to find the correlation coefficient and the least squares regression line in statistics.

Exercise: The following histogram gives the weights of 30 pupils in a class.



- 1) What type of data is it?
- 2) Prove that the frequency of the 1<sup>st</sup> class interval  $(40 \le m < 50)$  is 4. Then determine the frequency of each class interval from the graph.
- 3) Which graph do you have to draw to find the median? Find its value.
- 4) Interpret the median in the context of the exercise.
- 5) The heights of the 6 heaviest pupils are given in the following scatter graph.



Is there a correlation between the mass and the height of the pupils?

- 6) Use the line of best fit to guess the height of a pupil whose weight is 73 kg.
- 7) Can you predict the weight of a child who is 120 cm tall?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles Binôme : Anglais / Mathématiques

## STATISTICS Sujet D4-72

The first part is a summary that can help you do the exercise.

When repeated observations are made on a variable, the result is a frequency distribution. A frequency distribution is recorded in a frequency table. The frequency table gives the possible values or class intervals of the variable and the corresponding frequencies.

A histogram illustrates a frequency distribution. It consists of rectangles drawn on a continuous base. The area of each rectangle is proportional to the frequency of the class it represents. The frequency density is given by the formula:  $ncy \ density = \frac{frequency \ of \ class \ interval}{frequency}$ 

width of class interval

You find the cumulative frequency by adding each frequency to the sum of all preceding frequencies. Cumulative frequency provides a convenient way of estimating a median (when the distribution is split into 2 parts), quartiles (when the distribution is split into 4 parts) without considering the raw data.

The histogram below shows the distribution of house prices in an area of Manchester. Prices are given in thousands of pounds (to the nearest thousand).



1) Explain in your own words how you can complete this histogram (without drawing on the exercise).

2) Copy the table above on your draft paper and complete with the frequencies for each class interval.

3) Work out an estimate of the mean price of the houses. (Give your result to the nearest pound)

4) Work out the cumulative frequencies.

5) Draw the cumulative frequency graph on your draft paper using a scale of 1 cm for £50,000 on the x-axis and 1 cm for 20 houses on the y-axis.

6) Using this graph, give an estimation of the median and the interguartile range.

Explain precisely how you find these parameters.

7) How many houses are sold more than £400,000?

Adapted from IGCSE

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

### Binôme : Anglais / Mathématiques

#### STATISTICS Sujet D4 81

<u>Recap</u>: You can use your calculator to find the correlation coefficient: it allows you to justify the correlation between two variables. The calculator also gives the line of best fit which helps you to make predictions.

For a continuous data, you obtain the median thanks to a cumulative frequency curve: it is the x-coordinate of the point whose y-coordinate is half of the total frequency.

Exercise:

We want to study how the children of a family use their mobiles.

	Child 1	Child 2	Child 3	Child 4	Child 5			
age	7	7	8	12	15			
Number of SMS/day	10	15	16	30	35			

Thanks to a survey, we obtained the following table:

1) We are first interested in finding a link between the age of a child and of the number of SMS sent per day.

- a) Explain how you can draw a scatter graph to illustrate this table, and draw it on your draft paper.
- b) Is there any correlation between the age of a child and the number of SMS he/she sends per day? Justify.
- c) Explain how you draw the line of best fit on your scatter graph.
- d) How old would a child who sends 20 SMS per day be?
- e) Could you predict the number of SMS sent per day for someone who is 50 years old?
- 2) Calculate the mean number of SMS sent per day in this family.
- 3) Calculate the median number of SMS sent per day in this family.
- 4) Let's deal with John, the eldest child. We studied the number of photos taken per day in January. We obtained the following cumulative frequency curve.

Find the median number of photos taken per day and explain what it means.

 30

 20

 20

 10

 0

 10

 0

 10

 0

 10

 0

 10

 0

 10

 0

 10

 0

Cumulative frequency

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## STATISTICS

Sujet D4-82

#### Recap

A linear equation of a straight line is given by y = ax + b, where a is the slope and b the y-intercept.

The graph represents the winning time for the men's 100-m freestyle swimming for selected Olympic games.



Let *y* represent the winning time. Let *x* represent the number of years since 1948 : x = 0 corresponds to the year 1948, x = 4 corresponds to 1952, and so on.

- 1. Use the ordered pairs given in the graph (0; 57.3) and (48; 48.7) to find a linear equation to estimate the winning time for the men's 100-m freestyle in terms of the year. Round the slope to 2 decimal places.
- 2. Use the linear equation from question 1 to approximate the winning 100-m time for 1972, and compare it with the 1996 winning time.
- 3. Use the linear equation to approximate the winning time for the year 1988.
- 4. What is the slope of the line and what does it mean in the context of this problem?
- 5. Explain why the men's swimming times will never reach the x –intercept.
- 6. Do you think this linear trend will continue for the next 50 years, or will the men's swimming times begin to level off at some time in the future? Explain your answer.

## BACCALAURÉATS GÉNÉRAL ET TECHNOLOGIQUE Session 2018

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## STATISTICS

#### D4-83

## Exercise 1

In a survey, two groups of teenagers wrote down how many hours of television they watched in one week.

Here are the box-and-whisker diagrams for the two groups.



(a) Fill in the blanks in this sentence:

About half of the teenagers in group A watched TV at least ... hours that week, whereas about three quarters of the teenagers in group B watched TV no more than ... hours.

- (b) Write a similar sentence, comparing groups A and B, using the lower quartile of group A.
- (c) Work out the interquartile range of hours of TV for each group.
- (d) Did the teenagers in group B spend more time watching TV than in group A?

### Exercise 2

These are the scores of two players over six rounds of golf:

lan	87	69	80	86	84	81
William	77	91	90	85	67	71

Remember: the lower the score, the better the player.

- (i) Calculate the mean score for each player, to 1 decimal place.
- (ii) The standard deviations of the scores are  $\sigma_{lan} \approx 5.9838$  and  $\sigma_{William} \approx 9.1727$ . Round these standard deviations to the nearest whole number.

Use the data to state who the better player is. Give a reason for your answer.

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

## Statistics Sujet D4\_84

## Exercise 1

Two workers iron clothes. Each worker irons 10 items, and records the time it takes him to iron each item, rounded to the nearest minute:

Worker A: 3 5 2 7 10 4 5 5 4 12 Worker B: 3 4 8 6 7 8 9 10 11 9

- 1. For worker A, find:
  - a. the median
  - b. the lower and upper quartiles
- 2. For worker B, find:
  - a. the median
  - b. the lower and upper quartiles
- 3. Draw two box-and-whisker plots representing the workers' times (i.e. one box-andwhisker plot for each worker). Both box-and-whisker plots should be drawn using the same scale.
- 4. Make one statement comparing the two sets of data. Which worker is the most efficient?

*iron = repasser* 

## Exercise 2

In a supermarket, two types of chocolate drops are compared.

The total weight of 20 chocolate drops of brand A is 60.3g. The mean weight of 30 chocolate drops of brand B is 2.95g.

- 1. Find the mean weight of a chocolate drop of brand A.
- 2. Find the mean weight of all the 50 chocolate drops.

## BACCALAURÉATS GÉNÉRAL ET TECHNOLOGIQUE Session 2018

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# Advanced Geometry

Sujet D5 - 41

The first part of this page is a summary which can help you solve the following exercise.



#### **EXERCISE**

The diagram on the right shows three straight highways surrounding a city which is represented by the hatched area.

- **1.** What's the bearing of *C* from *A*?
- **2.** Calculate distance *AH* in terms of *x*.
- **3.** Calculate distance *AC* in terms of *x*.
- 4. The area of the city is about 1600 hectares (given that 1 hectare =  $10\ 000\ m^2$ ). Since a triangle made by the three highways must be larger than the area of the city, find the lowest possible value of *x* to the nearest metre.



**5.** Deduce the position of point *B* from point *H*. (To locate one point from another one, you must give the bearing of this point from the other and the distance between the two points).

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

## Binôme : Anglais / Mathématiques

Advanced Geometry Sujet D5 - 45

The first part of this page is a summary that can be useful to do the exercise.

If *ABC* is any right-angled triangle with *C* as the right angle, and the sides of lengths *a*, *b*, *c* are opposite the angles *A*, *B* and *C* respectively, then the trigonometric ratios are as follows:



 $\cos(A) = \frac{b}{c}$   $\sin(A) = \frac{a}{c}$   $\tan(A) = \frac{a}{b}$ 

For over 200 years, mathematicians have been attempting to find accurate values for  $\pi$  and whether this number has any pattern to its decimal form. An early approximation was given by Archimedes in about 250 BC who used 96-sided regular polygons to estimate the value of  $\pi$ . The activity which follows gives the method used by Archimedes.

#### Example : let's start with a square

Consider a circle of radius 1, its area is :  $\pi \times 1^2 = \pi$ . By considering its inscribed square and circumscribed square, it will give an approximation of  $\pi$ .





<u>The area of the inscribed square is :</u> The length of the side of a square is the length of the hypotenuse of the inscribed isosceles right triangle. Thus, the area of the inscribed square is :  $\sqrt{2} \times \sqrt{2} = 2$ 

<u>We can deduce the upper and lower bounds for  $\pi$ .</u> The area of a circle with radius 1 is equal to  $\pi \times 1^2 = \pi$ , so  $2 \le \pi \le 4$ 



Your task is to investigate an approximation of  $\pi$  using a 5-sided regular polygon (5 equal sides).

- a) Explain why  $\theta = 36^{\circ}$ .
- b) In triangle OAB, what are the lengths of the height OP, and the corresponding base AB?
- c) What is the area of the inscribed triangle AOB?
- d) What is the area of the circumscribed triangle ORQ?
- e) Deduce the upper and lower bounds for  $\pi$ .

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## ADVANCED GEOMETRY

Sujet D5 - 52

The first part of this page is a summary which can help you solve the following exercise.

An ordered pair (a,b) of real numbers is a **solution to an inequality in** x **and** y if the substitution x = aand y = b satisfies the inequality. For example, the ordered pair (2, 5) is a solution of y < 2x + 3 because  $5 < 2 \times 2 + 3$ , that's to say 5 < 7. However, the ordered pair (2, 8) is not a solution because  $8 > 2 \times 2 + 3$ , in other words, 8 > 7. The **graph of an inequality in** x **and** y consists of all pairs (x, y) that are solutions of the inequality. The graph of an inequality involving two variables is a region of the coordinate plane. The point (2, 7) is on the graph of the line y = 2x + 3 but is not a solution of y < 2x + 3. A point (2, y) below the line y = 2x + 3 is a solution of y < 2x + 3. The graph of y < 2x + 3 is the set of all points below the line y = 2x + 3.

### Exercise:

M. Gable builds models of famous buildings with matches. He needs 5 days and 300 thousand matches to build an Eiffel Tower and 3 days and 450 thousand matches to build a Big Ben Tower. M. Gable has 6 million matches in stock for his models and 60 days to build them.

An Eiffel Tower is sold for 50 pounds and a Big Ben Tower is sold for 70 pounds.

The goal of this exercise is to find how many models of each type M. Gable should sell in order to maximize his **turnover**<sup>1</sup> under the previous constraints.

Let x be the number of Eiffel Tower models built.

Let y be the number of Big Ben Tower models built.

Let T be the turnover in pounds for the sale of x Eiffel Towers and y Big Ben Towers.

Show that  $2x + 3y \le 40$ 

- **1)** Find another inequality in x and y.
- Using the following figure, shade the unwanted region corresponding to these inequalities.
- 3) Show that  $y = -\frac{5}{7}x + \frac{T}{70}$ .
- Draw line Δ corresponding to a turnover of 700 pounds.
- 5) Conclude using the graph.



Page 1/1

<sup>1</sup> Turnover: *Chiffre d'affaire* 

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

## ADVANCED GEOMETRY

Sujet D5 - 61

The first part of this page is a summary that can be helpful to do the exercise.

A linear function whose equation is ax + by = c is represented by a straight line dividing the Cartesian plane into two half-planes, one on each side of the line. One half-plane contains the solutions to the inequality ax + by < c, while the other contains the solutions to ax + by > c. If the points on the dividing line (also known as the "**boundary line**") are part of the solution set, we write the inequalities as  $ax + by \le c$  and  $ax + by \ge c$ . respectively.

Example:

The equation 2x + y = 4 is represented in the *xy*-plane by a sloping line.

The inequality 2x + y < 4 is represented by a **half-plane** bounded by this sloping line. To decide which half-plane represents the given inequality, we can use a check point that is not on the line. The easiest point to use is the origin. When x = 0 and y = 0, 2x + y = 0, which is less than 4. So the origin is in the region that represents 2x + y < 4.

A shop stocks only sofas and beds.

A sofa takes up 3 m<sup>2</sup> of floor area and is worth £600. A bed takes up 4 m<sup>2</sup> of floor area and is worth £300. The shop has 45 m<sup>2</sup> of floor space to stock.

The shop stocks at least 3 sofas and 2 beds at any one time.

The insurance policy will allow a total of only £6,000 of stock to be in the shop at any one time. The shop stocks x beds and y sofas.

- **1.** Explain why  $x + 2y \le 20$ .
- 2. Give three other inequalities in *x* and/or *y* to show this information.
- 3. Could the shop stock 7 beds and 6 sofas at any one time? Explain your answer using your inequalities.
- **4.** Using the diagram:
- **a.** Shade the unwanted region. **b.** Tell if it's possible for the shop to stock at any one time: (i) 4 beds and 4 sofas; (ii) 3 beds and 9 sofas. 5. Calculate the coordinates of A. Check your result graphically. 6. A bed is sold £800 and a sofa £1,000. a. Determine the equation of a line D corresponding to a revenue of £10,000 and draw the line on the graph. **b.** Is it possible for the shop to make £10,000? U

What is the maximum revenue the shop can make? Give the corresponding numbers of beds and sofas which have to be sold to make this revenue.
#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

### Binôme : Anglais / Mathématiques

Sujet D5 - 81

# **Advanced Geometry**

The first part of this page is a summary which may help you solve the following exercise.

In all cases  $(x_1, y_1)$  and  $(x_2, y_2)$  represent points. - Midpoint of a line segment:  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ - Distance *d* between two points:  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ - Slope *m* of a line, given two points:  $m = \frac{y_2 - y_1}{x_2 - x_1}$ 

# <u>Exercise</u>

An undersea tunnel is to be built from Wicklow, Ireland, to Morfa, Wales, as shown on the diagram. The tunnel will be 45 m below sea level at P but will gradually rise 5 m at point Q. The point S is 4.5 km from Wicklow and the point R is 4.8 km from Morfa. The distance from Wicklow to Morfa is 92.8 km.



Taking W as the origin (0,0) and using metres as units, find:

(i) the slope of WP, the slope of PQ and the slope of QM;

(ii) the equation of the line representing the tunnel from P to Q;

(iii) the total length of the tunnel from W to M, rounding to the nearest metre.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

### Binôme : Anglais / Mathématiques

# Sujet D5 – 82 D5- : Advanced Geometry

**Exercise:** The Tealicious company ships two types of containers on the Thames, containers with black tea and containers with green tea, under the following constraints:

- The weekly tea production of the company cannot exceed 1200 containers.
- Projections indicate an expected demand of at least 200 containers of black tea per week.
- The demand also requires the shipping of at least two containers of green tea for each container of black tea shipped.

The shipping of a container of black tea yields a profit of  $30 \text{ \pounds}$  whereas the shipping of a container of green tea yields a profit of  $20 \text{ \pounds}$ .

Let x be the number of black tea containers shipped per week. Let y be the number of green tea containers shipped per week.

- 1- Find three inequalities that must be satisfied by x and y.
- 2- Match each inequality from the first question to a line of the following graph and shade the region that <u>does not satisfy</u> all the constraints of the Tealicious company



- 3- Can the Tealicious company ship 400 containers of black tea and 600 containers of green tea?
- 4- Given that *P* is the profit for the shipping of *x* containers of black tea and *y* containers of green tea show that *P* satisfies:

$$y = -\frac{3}{2}x + \frac{P}{20}$$

5- Explain how to find the number of containers of each type that must be shipped every week in order to maximize the profit of the Tealicious company and calculate that maximum profit.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet D5 – 83

# Advanced Geometry

**Exercise:** On the following radar, we can see two airplanes A and B flying from and to Heathrow (H), one of the airports of London.



This radar is made of lines radiating at 10° intervals and of concentric circles with centre H. The radius of each circle is 10 miles greater than the former one. The radius of the smallest circle is 10 miles.

- 1. Explain how to find the bearing of airplane A from Heathrow.
- 2. Deduce the bearing of Heathrow from airplane A.
- 3. Find the distance between airplane A and airplane B rounding to the nearest mile.
- 4. An airplane C leaves Heathrow and flies for 45 minutes at an average speed of 120mph on a bearing of 320 degrees.
  - a. Plot the point T where the airplane turns on the radar.
  - b. Then it turns due West and continues another 45 minutes at the same speed. How far is airplane C from Heathrow?
  - c. What is the bearing of airplane C from Heathrow?

# Vocabulary:

One radius – two radii

Due West = Heading to the West

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet D5 – 84

# **ADVANCED GEOMETRY**

The first part of this page is a summary which may help you solve the following exercise.

An ordered pair (a,b) of real numbers is a **solution to an inequality in** x **and** y if the substitution x = a and y = b satisfies the inequality. For example, the ordered pair (2, 5) is a solution of y < 2x + 3 because  $5 < 2 \times 2 + 3$ , that is to say 5 < 7. The graph of an inequality involving two variables is a region of the coordinate plane. The point (2, 7) is on the graph of the line y = 2x + 3 but is not a solution of y < 2x + 3. A point (2, y) below the line y = 2x + 3 is a solution of y < 2x + 3.

# Exercise:

Bob's factory produces two types of pizzas: "xtra pizza" and "yankee pizza". Let x be the number of xtra pizzas which are made. Let y be the number of yankee pizzas which are made.



- **1.** Bob's production is subject to different constraints.
  - **a.** Two constraints are  $y \le 200$  and  $x \le 300$ . Explain what this means in terms of pizzas.
  - **b.** Explain which of the different regions of the above graph is the accepted one.
- 2. Each *xtra pizza* requires 5 mushrooms and 8 olives.

Each *yankee pizza* requires 10 mushrooms and 4 olives.

Bob's factory has to use at least 2500 mushrooms and at least 2400 olives.

- **a.** Write two further inequalities to represent this information.
- **b.** Add two lines and shadings on the diagram above to represent these two inequalities.
- **c.** Determine the feasible region and label it  $\mathcal{R}$ .
- **3.** The factory needs 10 minutes to produce each *xtra pizza* and 4 minutes to produce each *yankee pizza*. Bob wishes to minimize the total time *T* taken to produce the pizzas.
  - **a.** Write *T* in terms of *x* and *y*, then *y* in terms of *x* and *T*.
  - **b.** Find the optimal number of each type of pizzas that Bob should produce, and find the total time it will take.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet D5 – 85

# Advanced Geometry

A captain wants to sail his ship from port A to port B, but the journey cannot be made directly. Port B is 40 km North of A.

The ship sails 20 km on a bearing of 075° to reach point P. Then it sails 20 km on a bearing of 350° and it drops anchor in point S.



- 1) Plot points P and S on the given diagram.
- 2) Let A be the origin of an orthonormal coordinate plane 1 km unit.
  - *a.* Give the coordinates of points A and B.
  - b. Work out the coordinates of points P and S. Give values to 1 d.p.
- 3) Compute the distance between S and B and give the result to 1 d.p.

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

PROBABILITY Sujet D7-43

### **Pi's Pottery**

The first part of this page is a summary that can help you do the exercise.

Binomial distribution:

Suppose that a trial is repeated a number of times, say *n*, and that in each trial there are two possible outcomes *A* and *B*. If P(A) = a and P(B) = b = 1 - a, and the outcomes of every trial are independent from each other, the probability of obtaining *k* events A among the *n* trials is:

$$P(A \text{ occurring } k \text{ times}) = {n \choose k} a^k (1-a)^{n-k},$$

where  $\binom{n}{k}$  ("*n* choose *k*") is the number of paths leading to *k* successes in *n* experiments.

Pi's Pottery produces mugs decorated with the faces of famous mathematicians.

Three quarters of them are decorated with a picture of Isaac Newton and the rest are decorated with a picture of John Napier.

The daily production of mugs is called a batch.

**1.** A mug is taken at random. It is then replaced and a second mug is taken at random.

Work out the probability that:

a. the two mugs are decorated with a picture of Newton;

b. the two mugs are decorated with the same mathematician;

c. at least one mug is decorated with a picture of Newton.

You may use a tree diagram in this question.

**2.** On average, 5% of the mugs are defective. So, before sending a batch out to the shops, the company wants to inspect the production in order to detect defective mugs. There are two ways of carrying out this inspection:

**Complete inspection** involves inspecting each item that is produced.

**Sampling** is the process of selecting a few units from the batch and operating complete inspection on the sample.

**a.** Suppose that, on a particular day, a batch of 100 mugs is produced.

What is the probability that exactly 4 mugs are defective?

**b.** The company is considering adopting the following sampling plan: "select 10 mugs from the batch at random and accept the batch if there are 2 or less defective mugs, otherwise reject batch".

What is the probability that the batch will be accepted?

c. Why would a company choose one or the other of these two inspection methods?



### SESSION 2018

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »

Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

#### PROBABILITY

Sujet D7 - 51

The first part of this page is a summary that can be helpful to do the exercise.

#### Conditional probability

Let A and B be two events, with  $P(B) \neq 0$ .

The probability of *A* given *B* is:  $P(A/B) = \frac{P(A \cap B)}{P(B)}$ .

#### **Binomial distribution**

Suppose that a trial is repeated *n* times, and that in each trial there are two possible outcomes A and B. The trials are independent from each other.

With P(A) = p and P(B) = 1 - p:

$$P(A \text{ occurring } k \text{ times}) = \binom{n}{k} p^k (1-p)^{n-k},$$

where  $\binom{n}{k}$  ("*n* choose *k*") is the number of paths leading to *k* successes in *n* experiments.

### EXERCISE

 A circular board, with a radius of 10 cm, is placed on a wooden square which side measures 30cm. One <u>dart</u>\* is thrown by a player. The player is equally likely to hit anywhere within **the square**.

Show that the probability of hitting the board is p = 0.35 to 2 d.p.

**2.** Paul throws two darts. The probability for him to hit the board with his first dart is p = 0.35.

If he hits the board with his first dart, the probability for him to hit the board with his second is 0.4.

If he doesn't hit the board with his first dart, the probability he doesn't hit the board with his second dart is 0.85.

- **a.** Draw a tree diagram.
- **b.** Find the probability that Paul hits the board both with his first and second dart.
- **c.** Find the probability that Paul hits the board with his second dart.
- **d.** Given that Paul has hit the board with his second dart, find the probability he hits the board with his first dart.
- **3.** Stella doesn't get influenced easily: whether she hits the board or not, the probability she hits the board again remains p = 0.35.
  - a. What is the probability she hits the board exactly twice in six attempts?
  - **b.** What is the probability she hits at least once the board in six attempts?
  - **c.** How can you find the number of darts she should throw if she wants to hit the board at least once with a probability greater than 99%?
  - Dart = fléchette

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

### Binôme : Anglais / Mathématiques

#### PROBABILITY

#### Sujet D7-61

**Exercise:** The Tealicious Company owns jumbo hopper barges and uses them to ship cubic containers on the Thames. Each cubic container can contain either black tea or green tea and can be shipped either to London or to Oxford. The Tealicious Company warehouse contains 6,200 cubic containers, 1,860 of which contain black tea. 20% of the black tea containers should be sent to Oxford whereas 60% of the green tea containers should be sent to London. A cubic container is chosen at random in the warehouse.

Let L be the event "the cubic container is sent to London" and G the event "the cubic container contains green tea".

1.

- a. Show the data in a tree diagram and describe it.
- b. Show that  $P(L) = \frac{33}{50}$
- c. After the shipping, a container is picked at random in Oxford. What is the probability that it contains green tea?
- 2. 10 cubic containers are chosen independently in the warehouse. What is the probability that (give your results rounded to 4 decimal places):
  - a. 5 of them should be shipped to London?
  - b. At least two of them should be shipped to London?

### SESSION 2018

### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »

### Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

### PROBABILITY

Sujet D7 - 62

The first part of this page is a summary that can be helpful to do the exercise.

Let A and B be two events. P(A) (respectively P(B)) is the probability that event A (respectively event B) occurs.  $P(A \cap B)$  is the probability that both events A and B occur.  $P(A \cup B)$  is the probability that at least one of the two events A or B occurs ;  $P(A \cup B)=P(A)+P(B)-P(A \cap B)$ . If A and B are mutually exclusive,  $P(A \cup B)=P(A)+P(B)$ . P(A)+P(not A) = 1. P(A|B) (read "the probability of A given B") is a conditional probability. It is the probability that event A occurs given the occurrence of event B. The multiplication rule states that  $P(A \cap B) = P(A|B) \times P(B)$ .

On my way to work, I drive through two sets of roadworks with traffic lights which only show green or red. I know that the probability that the first is green is  $\frac{1}{3}$ . If the first is green, the probability that the second is green is  $\frac{1}{3}$ . If the first is red, the probability that the second is green is  $\frac{2}{3}$ .

1) Draw a tree diagram, showing the possible outcomes when passing through both sets of lights.

2) What is the probability of each of the following outcomes?

- a) I do not get held up at either set of lights.
- b) I get held up at exactly one set of lights.
- c) I get held up at least once.
- d) I get held up at the second set of lights.

3) Given that a car has been held up at the second set of lights, what is the probability that it hadn't been held up at the first?

4) Over a school term I make 90 journeys to work. On how many journeys can I expect not to get held up by a red light?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

### PROBABILITY

Sujet D7 - 63

The first part of this page is a summary that can help you do the exercise.

Let A and B be two events. P(A) (respectively P(B)) is the probability that event A (respectively event B) occurs. P(A') is the probability the complement event of A. P(A') + P(A) = 1  $P(A \cap B)$  is the probability that both events A and B occur. P(A|B) (read "the probability of A given B") is a conditional probability. It is the probability that event A occurs given the occurrence of event B. The multiplication rule states that  $P(A \cap B) = P(A|B) \times P(B)$ .

#### Exercise:

In the factory Toys'R'Maths, a machine produces wooden toys for children.

- When the machine operates well, the probability of producing good-quality toys is 0.99;
- When the machine has certain problems, the probability of producing goodquality toys is 0.51;
- Every morning, when the machine is started, the probability that the machine operates well is 0.95.

Let T be the event "the toy produced is a good-quality toy" and N the event "the machine operates well". T' and N' are the complement events of T and N, respectively.

- 1. Show the information above in a tree diagram.
- 2. Work out the probability of the event "the machine operates well and produces a good quality toy".
- 3. a. Explain what the event  $T' \cap N'$  is. Calculate  $P(T' \cap N')$ .
  - b. The CEO of Toys'R'Maths states that 97.55% of the production is goodquality toys. Is he right?
- 4. If on one morning, the first toy produced by the machine is of good quality, what is the probability that the machine operates well?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

### PROBABILITY

Sujet D7 - 65

The first part of this page is a summary that can help you do the exercise.

▶ Probability is the likelihood of an event to happen. If the outcomes of a random event are equally likely, the probability for any event to occur is:

total number of all possible outcomes

▶ Binomial distribution: Suppose that a trial is repeated a number of times, say *n*, and that in each trial there are two possible outcomes: *A* and *B*. If P(A) = a and P(B) = b = 1 - a, and the outcomes of every trial are independent from each other, the probability of obtaining *k* events *A* among the *n* trials are:

$$P(A \text{ occurring } n \text{ times}) = {n \choose k} a^k (1-a)^{n-k}$$

where  $\binom{n}{k}$  is the number of paths leading to k successes in n experiments.

• When X follows a binomial distribution B(n, p), the expected value of X is  $E(X) = n \times p$ .

### EXERCISE

A crossword puzzle is published in *The Times* each day of the week, except on Sundays. Jack is able to complete, on average, 8 out of 10 of the crossword puzzles.

The fact that Jack succeeds in completing a crossword puzzle on a given day is independent from his success or failure the other days.

- **1)** Let *X* be the number of crosswords Jack completes in a given week. Explain why *X* follows a binomial distribution B(6, 0.8).
- 2) Find the expected value of the number of completed crosswords in a given week.
- 3) What is the probability that Jack completes all the crossword puzzles in a given week?
- **4)** Show that the probability that Jack will complete at least 5 in a given week is 0.655 (to 3 significant figures).
- **5)** Given that Jack completes the puzzle on Monday, May 22<sup>nd</sup>, find the probability that he will complete 4 crosswords in the rest of the week. Give the result to 3 significant figures.
- 6) Find the probability that, in a period of four weeks, Jack completes 4 or fewer crosswords in only one of the four weeks. Give the result to 3 significant figures.

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

### PROBABILITY

Sujet D7-71

Prince Charming leaves for a dangerous adventure. He will have to deal with dragons (D) in 20% of cases, trolls (T) in 42% of cases and goblins (G) in every other situation. His probability to win (W) against dragons is 0.4 but he has twice the chance to win against trolls or goblins. If he loses, he dies.

Let W be the event "the prince wins". T' is the event "not T".

- 1) Draw a tree diagram and complete it.
- 2) Explain what the event T' means.
- 3) What is the probability that he will find goblins and win?
- 4) What is the probability that he will win the adventure?

5) Given that he has won, show that the probability that he has met a troll is about 0.47 (2 dp).

6) If he wins, the princess will be playing heads and tails to know if she has to accept his proposal. If she has heads, she will say yes, if she has tails she won't. But Prince Charming is smart and he has given her a biased coin so that the probability of having a tail is 0.15. What is the probability that they will get married?

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

# PROBABILITY

Sujet D7-72

The first part of this page is a summary that can be helpful to do the exercise.

•Two events A and B are independent if the outcome of one event does not affect the outcome of the other, in which case:  $P(A \text{ and } B) = P(A) \times P(B)$ .

• P(an event does not occur) = 1 – P(the event does occur).

• A simple tree diagram is a useful way to represent the probabilities of combined events.

# EXERCISE

A wheel is divided into twelve equal sectors numbered from 1 to 12.

**1.** Jack spins the wheel and looks at the number on the sector facing him. Let A be the event: "He obtains an odd number greater than 4";

Let B be the event: "He obtains a perfect square".

- (*i*) Find P(A) and P(B).
- (*ii*) Are A and B independent events?

**2.** Jack spins the wheel twice, independently.

- (*i*) Find the probability of the event:
  - "The first number he obtains is 9, and the second one is 11".
- (*ii*) Find the probability that the sum of the numbers he obtains is 20.
- (iii) Find the probability he obtains at least one 12.

**3.** Jack spins the wheel 10 times, independently.

Explain why the probability he obtains at least one 12 is 0.58 to two d.p.

**4.** How many times *n* should Jack spin the wheel for him to obtain at least one 12, with a probability greater than 99%?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

# PROBABILITY

Sujet D7-73

"You want to be careful with these. When they say every flavour, they mean every flavour - you know you get all the ordinary ones like chocolate and peppermint and marmalade, but then you can get spinach and liver and tripe. George reckons he had a bogeyflavoured one once. Ron picked up a green bean, looked at it carefully, and bit into a corner.

"Bleaaargh – see ? Sprouts."

J.K Rowling, Harry Potter and the Sorcerer's Stone, 1997

Harry buys a Bertie Bott's pack of jelly beans. The box contains a total of 40 sweets. Each sweet is either red, green or blue. You can get an ordinary flavor or a weird one.

The number of sweets of each type is shown in the table below:

	red	green	blue
ordinary	8	11	5
weird	6	8	2

1. Harry selects a bean at random, find the probability of it...

- A. ...tasting weird.
- B. ...not being blue.
- C. ...either tasting ordinary or being green.
- D. ...tasting weird, given that it's red.

2. Three beans are selected at random, without replacement, hopefully. Find the probability that two are weird and one is ordinary.

ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

#### PROBABILITY Sujet D7-81

Vocabulary:	<i>ocabulary: a bet</i> : un pari		to place bets on : miser sur
	a pocket / a slot : une case	-	<i>payout</i> : gain

Roulette is a casino game. Players choose to place bets on a single number **OR** a range of numbers **OR** on the colors red or black **OR** whether the number is odd or even.

To determine the winning number and color, a croupier spins a wheel in one direction, then spins a ball in the opposite direction.

The ball falls into one of 38 colored and numbered slots (marked from 1 to 36, plus 0 and 00) on the wheel.

There are 18 red slots, 18 black slots, and the "0" and "00" slots are green.



- 1) a) A player places a bet on number 4. What is his probability of winning?
  - b) For a single bet (a bet on only one single number), the payout is 35 to 1.
    It means that if you bet £1 on number 4 and 4 occurs, then you receive £1 + £35.
    If the ball doesn't fall into the slot "4", then you waste £1.
    Let *X* be the random variable equal to the amount the player wins or loses in one £1bet on a number. What are the possible values taken by *X*?
  - c) A player bets £1 on a number. Fill in the table below:

amount $(X = ?)$	
probability $P(X = ?)$	

- d) Work out the expected value of *X*. Give an interpretation of the result.
- 2) a) A player places a bet on "an odd number will occur". What is his probability of winning?

b) For an "even-odd bet ", the payout is 1 to 1. It means that if you bet £1 on "an odd number will occur" and if an odd number occurs then you receive £1 + £1. Otherwise, you wasted £1. Let *Y* be the random variable equal to the amount the player wins or loses in one £1 bet on "an odd number will occur ". What are the possible values taken by *Y*?

c) A player bets £1 on "an odd number will occur ". Fill in the table below:

amount $(Y = ?)$	
probability $P(Y = ?)$	

d) Work out the expected value of *Y*. Give an interpretation of this result.