

# **ANGLAIS / MATHÉMATIQUES**

SECTION EUROPÉENNE

SESSION 2022

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris – Créteil – Versailles

Binôme : Anglais / Mathématiques

Corrigé n°1

#### Thème : dérivation / Differentiation

1) The radius of the circle is the width of the rectangle. Therefore, the radius is x cm.

2) The perimeter is 60 cm. Then:  $x + 2 \times y + x + \frac{1}{4} \times 2\pi \times x = 60$ 

Thus 
$$2y = 60 - 2x - \frac{1}{2}\pi x$$
 and  $y = 30 - x - \frac{1}{4}\pi x$ 

3)  $A = x \times y + \frac{1}{4} \times \pi x^2 = x \left( 30 - x - \frac{1}{4} \pi x \right) + \frac{1}{4} \times \pi x^2 = 30x - x^2 - \frac{1}{4} \pi x^2 + \frac{1}{4} \pi x^2 = 30x - x^2.$ 

4) a) *A* is a quadratic function. The derived function is A'(x) = 30 - 2x.

The derivative is 0 when x = 15: for x = 15, *A* is stationary.

b)  $A(15) = 30 \times 15 - 15^2 = 225$ 

Furthermore, if x < 15 then 2x < 30 and A'(x) > 0 and if x > 15 then A'(x) < 0.

The sign of A'(x) is + on the left of 15 and – on the right, so the function A has a maximum value at x = 15 which is 225 cm<sup>2</sup>.

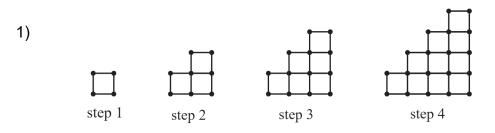
c) The dimensions of the plate are x = 15 cm,  $y = 15 - \frac{15}{4}\pi \approx$ 

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Corrigé n°2

Thème : suites/sequences



2)  $u_1 = 4$   $u_2 = 10$   $u_3 = 18$  $u_2 - u_1 = 6$  and  $u_3 - u_2 = 8$  therefore  $u_n$  isn't an AP.  $\frac{u_2}{u_1} = 2,5$  and  $\frac{u_3}{u_2} = 1,8$  therefore  $u_n$  isn't a GP.

Step n	1	2	3	4
$u_n$ = number of lines	4	10	18	28
$n^2$	1	4	9	16
$v_n = u_n - n^2$	3	6	9	12

4) a) v<sub>n</sub> seems to be an AP. The common ratio is 3 and the first term is v<sub>1</sub> = 3.
b) Therefore, v<sub>n</sub> = 3(n - 1) + 3 = 3n

5) a) 
$$u_n - n^2 = 3n$$
 then  $u_n = n^2 + 3n$   
b)  $u_{10} = 10^2 - 3 \times 10 = 70$   
c) Max will need  $S = u_1 + u_2 + \dots + u_{10}$  matches.  
 $S = 1^2 + 3 + 2^2 + 3 \times 2 + \dots + 10^2 + 3 \times 10 = (1^2 + 2^2 + \dots + 10^2) + 3(1 + 2 + \dots + 10)$   
 $= \frac{10(10+1)(2 \times 10+1)}{6} + 3\frac{10(10+1)}{2}$   
 $= \frac{10(10+1)(2 \times 10+1)}{6} + 3\frac{10(10+1)}{2}$   
 $= 550$ 

Max will need 550 matches to complete the 10 first steps of his pattern. He will need 3 boxes to do so.

 $240 \times 3 - 550 = 170$ : 170 matches will remain in the third box.

 $u_{11} = 11^2 - 3 \times 11 = 88$ : he could make the eleventh step with the remaining matches. Only 82 will still remain which won't be enough for another row.

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# MAPPING

Corrigé n°3

# <u>Answers</u>

1) No paraphrasing!

2) The new width is W + 11, the new length is L + 9, hence (W + 11)(L + 9) = 3WL and the perimeter of the initial rectangle is 2(W + L) = 58.

3) L = 29 - W and we replace L by this expression in the second equation.

4) After simplifying by 2,  $\Delta$  = 64 and the two solutions are 11 and 9.

Therefore, there are two possibilities:

If W = 11, L = 18 and the area is 198 mi<sup>2</sup>. If W = 19, L = 10 and the area is 190 mi<sup>2</sup>.

He got 198 mi<sup>2</sup>. The mayor had anticipated 190 mi<sup>2</sup>.

5) a) The side measures 2 miles, so the perimeter is equal to 8 miles.

b) 10 miles cost 500 000 dollars, so 1 mile costs 50 000 and 8 cost 400 000 dollars.

c) 400 000 x 0.85 = 340 000 dollars.

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Corrigé n°5

#### Thème : D3 – Suites – Sequences

1.  $u_0 = 12$   $u_1 = u_0 + \frac{50}{100}u_0 = 12 + \frac{12}{2} = 18$  $u_2 = u_1 + \frac{50}{100}u_1 = 18 + \frac{18}{2} = 27$ 

$$2.u_{n+1} = u_n + \frac{50}{100}u_n = u_n + 0.5u_n = 1.5u_n$$

- 3.  $(u_n)$  is a geometric progression.
- 4.  $(u_n)$  is a geometric progression. Its first term is  $u_0 = 12$ . Its common ration is r = 1.5.  $u_n = u_0 \times r^n = 12 \times 1.5^n$
- 5.  $u_4 = 12 \times 1.5^4 = 60.75$
- 6. a) diameter after 1 week?  $u_7 = 12 \times 1.5^7 = 205 cm$ (to 3 s.f.)

b) At this time,  $area = \pi R^2 = \pi \times (\frac{205}{2})^2 = 33000 cm^2$  to 2 s.f.

7. How long would it theoretically take for the water lily to cover a 10 meters wide pond?

10m = 1000cm

 $u_{10} \approx 692$  and  $u_{11} \approx 1038$ . It would take 11 days.

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Corrigé n°8

- **1)**  $0 \le x \le 6$
- **2)** x(6-x)
- 3) a parabola
- 4) The image of 5 is 5.
- 5) The preimages of 8 are 2 and 4.
- **6)**  $x(6-x) = 6x x^2$

7) 
$$-\frac{b}{2a} = \frac{-6}{-2} = 3$$
 and  $f(3) = 9$ .

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Corrigé n°9

1) a) Elimination or substitution

 $\begin{cases} 3x + 2y = 13 \\ 2x + y = 8 \end{cases} \qquad \begin{cases} x = 3 \\ y = 2 \end{cases}$ 

b) x is the number of scones and y is the number of muffins. Same simultaneous equations as 1a). Scone: £3 and Muffin: £2.

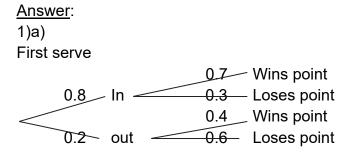
**2)** a) 3:1:1 = 600:200:200

200g of sugar and 200g of butter b) Total amount of paste: 600+200+200 = 1000 g 1000/200 = 5 He will cook 5 muffins.

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Corrigé n°10



1)b) P(wins point)=P(wins point and first serves in) + P(wins point and first serves out) =0.8\*0.7+0.2\*0.4=0.64

1)c) P(first serve out |wins)= $\frac{P(first \ serves \ out \ and \ wins)}{P(wins)} = \frac{0.2*0.4}{0.64} \approx 0.125$ 

2) a)and b) a trial = a point

Success = he wins

P(success)=0.57

We repeat 60 times the same trial independently

Let label X the random variable which gives the number of successes, X follows the binomial distribution with parameters 60 and 0.57

$$P(X = 40) = \binom{60}{40} * 0.57^{40} * 0.43^{20} \approx 0.03$$

2)c) E(X)=60\*0.57≈34

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Corrigé n°11

#### Answer:

1)  $h(x) = ax^{2} + bx + c$  h(0) = 2 then c=2, so  $h(x) = ax^{2} + bx + 2$  h(1) = 4.25 gives a + b + 2 = 4.25 then a + b = 2.25h(2) = 5 then 4a + 2b + 2 = 5 so 4a + 2b = 3

Simultaneous equations to solve by substitution or elimination

By substitution: a = 2.25 - b then 4(2.25 - b) + 2b = 3 so b = 3

Finally  $a = 2.25 - 3 = -0.75 = -\frac{3}{4}$  so  $h(x) = -\frac{3}{4}x^2 + 3x + 2$ 

2) we can find x such as h(x)=0

 $\Delta = 3^2 - 4 * (-0.75) * 2 = 15 > 0$  then two real roots:

$$\frac{-3 - \sqrt{15}}{2 * \left(-\frac{3}{4}\right)} \approx 4.58 \text{ and } \frac{-3 + \sqrt{15}}{2 * \left(-\frac{3}{4}\right)} \approx -0.58$$

As -0.58<0, the piece of bread lands in the water, at 4.58m from John and 42cm from the swans on the other river bank.

3) The x-coordinate of the vertex is  $-\frac{b}{2a} = -\frac{3}{2*(-\frac{3}{4})} = 2$ ,

then the greatest height is h(2)=5m

4) S(x)= mx+p

$$m = \frac{0 - 0.5}{3 - 0} = -\frac{1}{6} \text{ so } S(x) = -\frac{1}{6}x + p$$

But S(0)=0.5 so p=0.5

Finally  $S(x) = -\frac{1}{6}x + \frac{1}{2}$ 

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Corrigé n°13

# Answer:

# Part A

- 1) 249:195 = 81:65 (we have divided by 3)
- 2) 100-58= 42 so 42% of women.

$$\frac{42}{100}$$
 \* 249 = 104,58  $\approx$  105 women

3)  $\frac{91}{100} * 249 = 226,59 \approx 227$ 227 injured while riding an electric scooter.

249-227 = 2222 pedestrians injured by an electric scooter.

- 4) 40% (head injuries) + 32% (bone fractures) + 28% (contusions, sprains and lacerations) = 100% So NO other types of injuries.
- 5) Number of injured riders: 227

Number of injured riders with no helmet : 227 -10 = 217 217/227 \* 100  $\approx$  96%

Part B up to you!

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Corrigé n°16

#### Part A

1. $\begin{cases} f(10) = 120 \\ f(20) = 210 \\ f(90) = 0 \end{cases} \begin{cases} 100a + 10b + c = 120 \\ 400a + 20b + c = 210 \\ 8100a + 90b + c = 0 \end{cases}$
$2.\begin{cases} 100a + 10b + c = 120 \\ 400a + 20b + c = 210 \\ 8100a + 90b + c = 0 \end{cases} \begin{array}{c} -2R_1: -200a - 20b - 2c = -240 \\ R_2:  400a + 20b + c = 210 \\ 200a - c = -30 \end{cases}$
$\begin{cases} 100a + 10b + c = 120 & -9R_1: -900a - 90b - 9c = -1080 \\ 200a - c = -30 & R_3: & 8100a + 90b + c = 0 \\ 8100a + 90b + c = 0 & 7200a - 8c = -1080 \end{cases}$
$\begin{cases} 100a + 10b + c = 120 & -8R_2 & -1600a + 8c = 2400 \\ 200a - c = -30 & R_3: & 7200a - 8c = -1080 \\ 7200a - 8c = -1080 & 5600a = -840 \end{cases}  \text{donc } a = \frac{-840}{5600} = -0.15$
$\begin{cases} 100a + 10b + c = 120\\ 200 \times (-0.15) - c = -30\\ a = -0.15 \end{cases} \begin{cases} 100a + 10b + c = 120\\ 200 \times (-0.15) - c = -30\\ a = -0.15 \end{cases} \begin{cases} 100 \times (-0.15) + 10b + 0 = 120\\ c = 0\\ a = -0.15 \end{cases}$
$\begin{cases} b = 13.5 \\ c = 0 \\ a = -0.15 \end{cases}$

Part B:

- 1.  $f(30) = 0.15 \times 30 \times (90 30) = 270$ . There are 270,00 patients 30 days after the first cases.
- 2.  $f(t) = 0.15 \times t \times (90 t) = -0.15t^2 + 13.5t$  $\frac{-b}{2a} = \frac{-13.5}{2 \times (-0.15)} = 45$  and  $f(45) = 0.15 \times 45 \times (90 - 45) = 303.75$ . The maximum number of patients is 303,750 patients. It occurs 45 days after the first cases.
- 3. We want to find t such that  $f(t) \ge 270$ We solve the inequation :  $-0.15t^2 + 13.5t \ge 270$ , i.e.  $-0.15t^2 + 13.5t - 270 \ge 0$ .  $\Delta = 13.5^2 - 4 \times (-0.15) \times (-270) = 20.25$  $x_1 = \frac{-13.5 - \sqrt{20.25}}{2 \times (-0.15)} = 60$  and  $x_2 = x_1 = \frac{-13.5 + \sqrt{20.25}}{2 \times (-0.15)} = 30$ As the parabola opens downwards,  $f(t) \ge 270$  for t between 30 and 60. The hospital is under stress for 30 days.

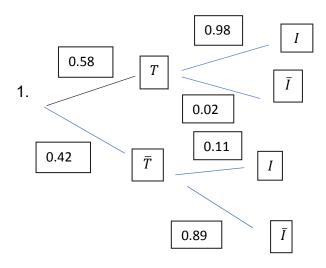
4. h = 303.75 and b = 90 - 0 = 90,

thus the area under the parabola is :  $\frac{2}{3}b \times h = \frac{2}{3} \times 90 \times 303.75 = 18,225$ . And the average number of patients during the epidemic was  $\frac{18,225}{90} = 202.5$  in thousands, Thus the average number of patients during the epidemic was 202,500 per day

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Corrigé n°17



- 2. a.  $P(T \cap I) = 0.58 \times 0.98 = 0.5684$
- b.  $P(T \cap \overline{I}) = 0.42 \times 0.11 = 0.0462$
- c. P(T) = 0.5684 + 0.0462 = 0.6146
- 3. specificity:  $P_I$  (*T*) =  $\frac{P(I \cap T)}{P(T)} = \frac{0.5684}{0.6146} \approx 0.92$  to 2 d.p.

Sensitivity :  $P_{\bar{I}}$   $(\bar{T}) = \frac{P(\bar{I} \cap \bar{T})}{P(\bar{T})} = \frac{0.42 \times 0.89}{1 - 0.6146} \cong 0.97$  to 2 d.p.

4. The test doesn't meet the criteria because the specificity is not high enough.

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Corrigé n°18

A. After one year there will be  $2000 \times 1.075 = \$2150$ . After two years there will be  $2150 \times 1.075 = \$2311.25$ . This last calculation can also be written as  $2000 \times 1.075 \times 1.075 = 2000 \times 1.075^2$ . Hence, after three years there will be  $2000 \times 1.075^3$  dollars and so on. After 8 years there will be  $2000 \times 1.075^8 \approx \$3566.96$ , to the nearest cent. Thus the final balance is approximately \$3566.96.

**Note:** The bank has paid \$1566.96 in interest and this corresponds to a simple interest rate of  $(1566.96 \div 2000) \times 100 \div 8 = 9.7935\%$  per annum.

B. Writing the amounts from smallest to largest, the total value of my investment will this be:

 $A = \$3000 \times 1.06 + \$3000 \times 1.06^2 + \dots + \$3000 \times 1.06^{30}$ 

This is a geometric series with  $a = \$3000 \times 1.06$ , and s = 1.06 and n = 30.

Substituting into the formula, the value of the sum is

$$A = \frac{3000(1.06^{30} - 1)}{1.06 - 1} \approx \$237,174.56$$

So, following this scheme \$90,000 invested becomes roughly \$237,000 over 30 years.

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# Corrigé n°19

Elements of correction:

- 1. At the 17<sup>th</sup> century, some of the mathematical problems under research were:
  - a. How to calculate the slope of a tangent line to a non-straight curve
  - b. Finding the maximum and minimum of functions
  - c. Finding the area under a curve
  - d. Finding the lengths of curves
  - e. Determining the curvature of a curve
  - f. Etc..

Sir Isaac Newton (1643-1727) lived in England. Studied natural science at Cambridge (Aristotle, Descartes, Galileo). Due to the bubonic plague (1665-1666), he goes back to live to the countryside for 2 years. He develops calculus and gravitational law (falling apple...).

Major works:

- 1687 Principia (*Principles of Natural Philosophy*); mechanics, gravitation, tides, planetary orbits, fluid dynamics, ... Needed the calculus.
- 1704 Opticks; refraction, reflection (also fluxions). Invented the Newtonian telescope
- 1707 Arithmetica Universalis (Universal Arithmetic)
- 1736 Method of Fluxions (written 1671)

Newton (and others) looked at mathematical problems as if they involved *dynamic, changing quantities*. For example, when studying a curve they considered a point's *x* and *y* coordinates changing as it moved along it. From this they could discuss the curvature of the curve and tangent lines to the curve (among other things).

Some terminology used by Newton:

- Fluent: something that changes ('moves'), e.g. points, lines, planes
- Fluxion: the velocity at which the fluent is moving
- Moment: the amount a fluent changes in a small amount of time due to its fluxion: moment = fluxion × time

The Leibniz–Newton calculus controversy:

Calculus, known in its early history as infinitesimal calculus, is a mathematical discipline focused on limits, continuity, derivatives, integrals, and infinite series. Isaac Newton and Gottfried Wilhelm Leibniz independently developed the theory of infinitesimal calculus in the later 17th century. By the end of the 17th century, both Leibniz and Newton claimed that the other had stolen his work, and the Leibniz–Newton calculus controversy continued until the death of Leibniz in 1716.

# 2. Exercice correction:

- a. Taking derivatives of  $x(t) = 12t^2 2t^3$  we obtain the velocity and the acceleration functions:  $v(t) = 24t 6t^2$  and a(t) = 24 12tPlugging in the value t = 3 yields v(3) = 18 m/s and  $a(3) = -12 m/s^2$
- b. At the maximum x, we must have v = 0; eliminating the t = 0 root, the velocity equation reveals  $t = \frac{24}{6} = 4s$  for the time of maximum x. Plugging t = 4 into the equation for x leads to x = 64m for the largest x value reached by the particle. We see that the x reaches its maximum at t = 4.0 s.
- c. A maximum *v* requires a = 0, which occurs when  $t = \frac{24}{12} = 2.0 s$ . This, inserted into the velocity equation, gives v = 24m/s. From this, we see that the maximum of *v* occurs at  $t = \frac{24}{12} = 2.0 s$ .
- d. The particle was (momentarily) motionless when x reaches its maximum at t = 4.0 s. The acceleration at that time is readily found to be  $24 12 \times 4 = -24 \text{ m/s}^2$ .
- e. The calculate the average velocity between t = 0s and t = 3s the values of x(t) at these values of t are needed; these are, respectively, x = 0 and x = 54m (found in part (a)). Thus, the average velocity is,

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{54m - 0}{3s - 0} = 18m/s.$$

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Corrigé n°21

Corr D0

1) a) False

b) True

c) True

- d) True (D=2r), we fine 2\*Pi\*r
- 2) a) They are isosceles triangles at O because OC = OB = OA (radii)
  - b) basis angles are equal:  $\Box$ BCO =  $\Box$ CBO and  $\Box$ BAO =  $\Box$ ABO.
  - c) The sum is equal to 180° so:

$$\alpha + \beta + (\alpha + \beta) = 180$$
  
2 (\alpha + \beta) = 180  
(\alpha + \beta) = 90  
20° it is a right angle

So □ACB = 90°, it is a right angle d) ABC is a right (or right-angled) triangle.

3) a) □BAC = 90°- 62° = 28° □BOC = 180° - 62° - 62°= 56° We notice that □BOC = 2\* □BAC

b) ABC is a right triangle from Thales' theorem. From Pythagoras' theorem,  $AB^2 + BC^2 = AC^2$   $6^2 + 8^2 = AC^2$   $36 + 64 = AC^2$   $AC^2 = 100$ so AC = 10, the diameter of its circumscribed circle is 10.

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Corrigé n°22

Correction D1

a)  $(x-a)^2 - 3 = x^2 - 2ax + a^2 - 3$ Therefore, a=1 b) disc=  $11^2 - 4 \times 2 \times (-6) = 121 + 48 = 169$  $\sqrt{169} = 13$ root1 =  $\frac{-11+13}{2\times 2}$  and root 2 =  $\frac{-11-13}{2\times 2}$ root1 =  $\frac{1}{2}$  and root 2 = -6  $\frac{1}{2} \times -6 = -3$  so b= -3 c) disc=  $2^2 - 4 \times 3 \times 1 = 4 - 12 = -8$ So c = 0d)  $xmin = \frac{-18}{2 \times 1} = -9$  $d(-9) = (-9)^2 + 18 \times (-9) + 85$ d(-9) = 81 - 162 + 85d(-9) = 4So d = 4e) x-3 = -3x+214x = 24x = 6y = 6-3=3So e = 3f)  $x^2 + 6x + 10 = 2$  $x^2 + 6x + 8 = 0$  $disc = 6^2 - 4 \times 1 \times 8 = 36 - 32 = 4$  $\sqrt{4} = 2$  $S1 = \frac{-6+2}{2} = -2$  and  $S2 = \frac{-6-2}{2} = -4$ 

The greatest is -2 so f=-2

1	-4	3
2	0	-2
-3	4	-1

Conclusion: Magic square

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Corrigé n°23

Thème : D0 – Core knowledge

1. Let's label PQ=x. Area of PQRS = PQ x QR = 10 x. So 45=10x and x=4.5.

In the right-angled triangle ABC, BC=4.5, AC=9.6. So  $AB^2 = AC^2-BC^2 = 9.6^2-4.5^2=71.91$ .

Thus AB=8.5, correct to 1 d.p. Finally the perimeter of ABCD is 2AB + 2BC=17+9=26.

Since AB is given correct to 1 d.p., then this result is also correct to 1 d.p.

**2.** 1 liter =  $1000 \text{ cm}^3$ . Total volume of the container =  $30x6x19=3420 \text{ cm}^3$ .

2/3 of the total volume is 2280 cm3. So the total amount of water in this container is 2.28 liters.

A cup holds 0.275 liter of water. Hence the number of cups that can be filled from this container is 2.28/0.275=8.3, correct to 1 d.p. So 8 full cups can be filled, which amounts to 8x0.275=2.2 liters. Finally, the remaining amount of water would be 2.28-2.2 = 0.08 liter = **80 ml**.

**3.** a) First, let's calculate the position of point M on DA. DM : MA = 2 : 3 means that DA represents 2+3=5 parts. Since DA = 15 cm, then one part is 3 cm. Finally : DM = 2x3=6 cm and MA = 3x3=9 cm.

**b)**  $\cos(35^{\circ}) = AB/AE$  so  $\cos(35) = 15/AE$  then  $AE = 15/\cos(35) \approx 18.3 \ cm$  to 1dp

c) Pythagoras th:

EM<sup>2</sup> = AM<sup>2</sup>+AE<sup>2</sup> = 9<sup>2</sup>+18.3<sup>2</sup> = 415.89

Finally, EM  $\approx$  20.4 cm

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Corrigé n°24

# Sujet n°Voillaume-D1 (corrigé)

- 1. 20*t*.
- 2. The second cyclist leaves 1 hour later, so the time he takes for the journey is 1 hour less than the time taken by the first cyclist.
- 3. Using Pythagoras Theorem, we have :

 $(20t)^{2} + (40(t-1))^{2} = 100^{2}$  $400t^{2} + 1600(t-1)^{2} = 10000$  $t^{2} + 4(t-1)^{2} = 25$  $5t^{2} - 8t - 21 = 0$ 

4. Using the quadratic formula, we find t = 3 hours.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Corrigé n°27

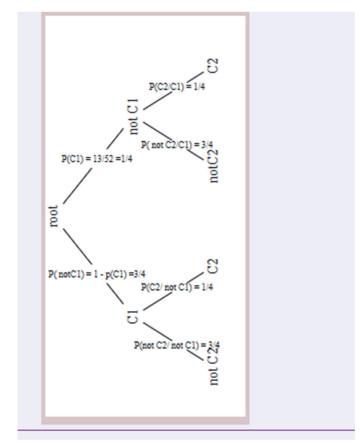
Sujet D00 correction

Let us call  $C_1$  the event : « the card is a club on the first throw »

Let us call Let us call  $\overline{C1}$  the event the : « card is not a club on the first throw »

Let us call C<sub>2</sub> the event : « the card is a club on the second throw »

Let us call  $\overline{C2}_1$  the event : « the card is a not club on the second throw »



1°) a) We are asked to compute

 $P(C1 \text{ and } C2) = P(C2/C1) \times P(C1) = 1/4 \times 1/4 = 1/16$ 

b) We are asked to compute

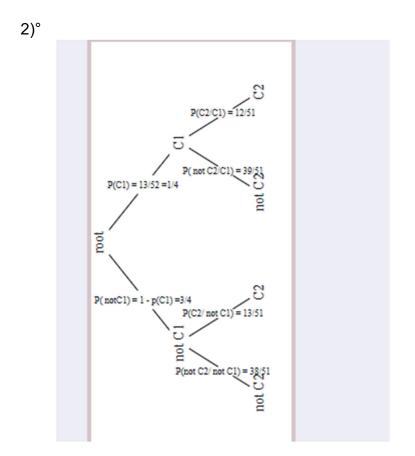
 $P(\text{not C1 and not C2}) = P(\text{not C2 / not C1}) \times P(\text{notC1}) = 3/4 \times 3/4 = 9/16$ 

c) We are asked to compute

P(C1 and not C2) + P(not C1 and C2) = P(not C2 / C1) × P(C1) + P(C2/ not C1) × P(notC1) =  $3/4 \times 1/4 + 1/4 \times 3/4 = 6/16 = 3/8$ 

d ) We are asked to compute

P(C1 or C2) = 1 - p(not C1 and not C2) = 1-9/16 = 7/16



1°) a) We are asked to compute

 $P(C1 \text{ and } C2) = P(C2/C1) \times P(C1) = 12/51 \times 1/4 = 1/3$ 

b) We are asked to compute

P(not C1 and not C2) = P(not C2 /not C1) × P(notC1) = 3/4 × 38/51 = 19/34

c) We are asked to compute

P( C1 and not C2 ) + P(not C1 and C2) = P(not C2 / C1 ) × P(C1 )+ P(C2/ not C1) × P(notC1) = 1/4 × 39/51 + 3/4×13/51 = 13/34

d ) We are asked to compute

P(C1 or C2) = 1 - p(not C1 and not C2) = 1-19/34 = 15/34

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

Corrigé n°28

1. Explain how the different verses of the song develop and describe the evolution on the number of present given each day and complete the following table

According to the text Christmas lasts 12 days. As time passes the number of gifts received increases by one.

n	1	2	3	4	5	6	7	8	9	10	11	
Un	1	2	3	4	5	6	7	8	9	10	11	
Vn	1	3	6	10	15	21	28	36	45	55	66	78

a) Could un be an arithmetic sequence? Give its characteristics.

# As the differences between two consécutive terms are all equal to one, $u_n$ seems to be an arithmetic sequence with an common difference d = 1 and a first term $u_1$ = 1.

b) Express  $u_{n+1}$  in terms of  $u_n$  and then the general expression of  $u_n$  ...

u<sub>n+1</sub> = u<sub>n</sub> + 1

 $u_n = u_1 + (n - 1) \cdot 1 = 1 + (n - 1) = n$ 

c) compute the nupmber of presents received on the twelfth day.

# u<sub>12</sub> = 12

c) Give the expression of  $v_n$  in terms of  $u_1$ ,  $u_2$ ,  $u_3$ ,..., $u_n$ , give the expression of  $v_n$  in terms of n and computev the total number given for Christmas.

 $v_1 = u_1 + u_2 + u_3 + \dots + u_n = \frac{(number of terms)(first term + last term)}{2} \\ = \frac{n(n+1)}{2} \\ We asked to compute \quad v_{12} = 12 (13) / 2 = 78$ 

2) Describe Pascal's triangle; explain how you can build it step by step, starting from the three one at the top ; fill in the last line.

n						
0	1					
1	1	1				
2	1	2	1			
3	1	3	3	1		
4	1	4	6	4	1	
5	1	5	10	10	5	1

To build Pascal's triangle one uses Pascal's formula

 $\binom{n+1}{p} = \binom{n}{p} + \binom{n}{p-1}$ 

 $\binom{n}{n}$  is a binomial coefficient and is read p choose n or p out of n.

The number that is below two other numbers is the sum of those two numbers. For instance

 $\binom{4}{2} = \binom{2}{2} + \binom{2}{1}$  gives 6 = 3 + 3

Step 1 : fill in the second column with ones Step 2 : fill in the diagonal with ones

Step 3 : fill in the missing lower blanks using the previous formula

The last line is 6 1 6 15 20 15 6

3) Let us ignore the second column of the triangle, which is filled with ones. Notice that the third column of the Pascal's triangle gives the the number of **new** presents given on the consecutive days (1, 2; 3, ...)

a) : Give the number of presents given on each day of the first five days

# First day :1 present; second day: 3 ; third day : 6 ; fourth day : 10 ; fifth day 15

b) Explain why the fourth column of Pascal's triangle indicates the total number of presents given on each day of the consecutive days.

The third column of Pascal's triangle indicates the number of new presents given on the consecutive days (1,2,3,...)

For instance, the number 10 in the fourth column is obtained by adding the number 6 (just above it on the same column) representing the number of presents given on day 3, and the number 4( just to the left of that 6) representing

We can assume that the total number of presents given on the n<sup>th</sup> is equal to  $\binom{n+1}{2}$ 

c) Compute with this formula the total number of presents given on the 12<sup>th</sup> day and check your result with question 1

So the total number presents given on the twelfth day is  $\binom{13}{2} = 13 \times 12 \div 2 = 78$ 

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Corrigé n°31

Topic: The British monarchy itself is one of the longest established monarchies in the world, and the Queen is believed to be related by blood or by marriage to every English king or queen since at least the 13th century.

The length of reign of each of the last 19 monarchs is given in the table.

George VI Edward VIII	16 years George IV 0 year George III	10 years <b>James II</b> 60 years	3
George V	26 years years	Charles II	25 years
EdwardVII	9 years George II	33 years Charles I	24
Victoria	64 years George I	13 years	
William IV	7 years years	James I	22 years
	Anne	12 years Elizabeth I	45
	William III	14 years years	
		Mary	5 years
		Edward VI	6 years

1) Order the data set from the lowest to the greatest value.(You can represent the data in an ordered stem and leaf diagram).

0-3-5-6-7-9-10-12-13-14-16-22-24-25-26-33-45-60-64

2) Find the median and quartiles of the length of reign of these 19 monarchs. You **must** show calculations to support your answer.

19 is an odd number. There is a middle number. When the data points are ordered from the lowest to the greatest value, the median is the 10 th value. The median is 14.  $Q_1 = 7$  (the 5 th value)

# $Q_3 = 26$ (the 15 th value)

3) Calculate the range and the interquartile range.

64 - 0 = 64 The range is equal to 64 years

#### $Q_3$ - $Q_1$ = 26 - 7 = 19 . The interquartile range equals 19 years.

4)Write down the name of any monarch whose length of reign is an **outlier**.

A box plot is constructed by drawing a box between the upper and lower quartiles with a solid line drawn across the box to locate the median. The following quantities (called *fences*) are needed to identify extreme values in the tails of the distribution:

- 1. lower inner fence: Q1 1.5\*IQ
- 2. upper inner fence: Q3 + 1.5\*IQ

A point beyond an inner fence on either side is considered a mild outlier. A point beyond an outer fence is considered an extreme outlier.

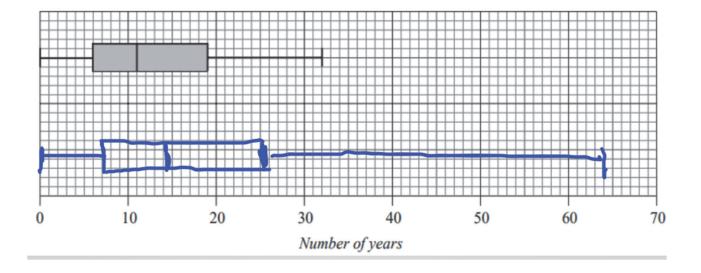
Q1 - 1.5\*IQ = 7-28,5 < 0

Q3 + 1.5\*IQ=26+28,5= 54,5

The names of any monarch whose length of reign is an outlier are George III and Victoria

5) The box and whisker plot shows the length of reign of the last 19 popes.

Draw a box and whisker plot for the length of reign of the last 19 monarchs on a copy of the diagram.



5) Are the statements true or false ?Explain your reasoning. Statement 1 : 25 % of the reigns of the last 19 popes are greater than 19 years

It is true because  $Q_3 = 19$ 

Statement 2: 50 % of the reigns of the last 19 popes are less than 11 years It is true because  $Q_1 = 11$ 

Statement 3 : 20 % of the reigns of the last 19 monarchs are greater than 20 years It is false because  $\frac{8}{19}$  > 20 %

6) Compare the length of reign of monarchs ans popes.

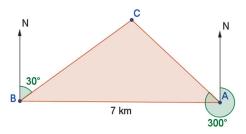
In the box plot for lenght of reign of the last 19 popes, the median (the vertical line inside the box) is slightly above 11 years, whereas the median for the lenght of reign of the last 19 monarchs is 14 years.

	Length of reign of the last 19 monarchs	Length of reign of the last 19 popes
Lowest value	0	0
$Q_1$	7	6
Median	14	11
$Q_3$	26	19
Greatest value	64	32
Range	64	32
Interquartile range	19	13

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Corrigé n°32



1) Work out the measures of angles  $\angle A$ ,  $\angle B$  and  $\angle C$  in the *ABC* triangle.

 $\angle A = 90^{\circ} - (360^{\circ} - 300^{\circ}) \qquad \angle B = 90^{\circ} - 30^{\circ} \qquad \angle C = 180^{\circ} - (30^{\circ} + 60^{\circ}) \\ \angle A = 30^{\circ} \qquad \angle B = 60^{\circ} \qquad \angle C = 90^{\circ}$ 

- 2) What can you say about triangle ABC? This is a right-angled triangle.
- 3) Work out the bearing of Tower B from Tower A . 270°
- 4) Calculate the distance between Tower *B* and the fire using the sine rule.

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin(\angle C)}$$
$$\frac{a}{(\sin 30^\circ)} = \frac{7}{(\sin 90^\circ)}$$
$$\boxed{a = \frac{7 \times \left(\frac{1}{2}\right)}{1} = 3.5}$$

5) Calculate the distance between *A* and *C*. Round to 2 dp.

Pythagoras theorem  $AB^2 = BC^2 + CA^2$  then  $AC \approx 6.06$ 

6) A fire-fighting plane can fly from tower *A* to the fire at a speed of  $303 \ km. \ h^{-1}$ . A fire truck can go from tower *B* to the fire at a speed of  $70 \ km. \ h^{-1}$ . Which vehicle will reach the fire first?

Fire truck :  $v = \frac{d}{t}$  then  $t = \frac{d}{v} = \frac{3.5}{70} = 0.05h = 3 \text{ minutes}$  Plane :  $v = \frac{d}{t}$  then  $t = \frac{d}{v} = \frac{6.06}{303} = 0.02h = 72 \text{ s}$ 

The plane will reach the fire first.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

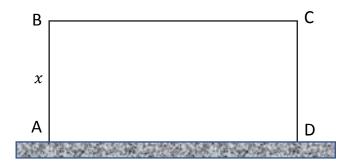
Binôme : Anglais / Mathématiques

Corrigé n°35

# DIFFERENTIATION

# Exercise 1

Emmy wants to build a rectangle enclosure for her animal (a goat) with a surface of 800 square feet. In order to minimize the coasts, she plans to build it against a wall of her house and wonders which minimum length of barrier she has to buy to surround the enclosure on the sides. She drew the figure below.



1. Let's call *x* the length AB. Explain how to find the expression of the length BC then compute the expression of the barrier's length.

The area of the enclosure which is a rectangle is 800 sq.ft so :

$$AB \times BC = 400 \Leftrightarrow BC = \frac{800}{AB} \Leftrightarrow BC = \frac{800}{x}$$

The barrier's length is:  $L = AB + BC + CD \iff L = 2x + \frac{800}{x}$  because as ABCD is a rectangle, AB = CD.

2. Let *f* the function defined by  $f(x) = 2x + \frac{800}{x}$ . What does represent this function? What should be the range of the study?

We notice that f(x) = L so the function f represents the length of the barrier. As x is a length, it can't be negative so we can study the function on  $]0; +\infty[$  (it seems difficult to consider that x could be less than 1 foot so a study on [1; 800] is convenient). 3. Work out f', the derivative function of f, and find the sign of f' to determine where the function is increasing and where it is decreasing.

$$\forall x \in ]0; +\infty[, f'(x) = 2 - \frac{800}{x^2}$$
$$\Leftrightarrow f'(x) = \frac{2x^2 - 800}{x^2}$$
$$\Leftrightarrow f'(x) = \frac{2(x^2 - 400)}{x^2}$$
$$\Leftrightarrow f'(x) = \frac{2(x^2 - 400)}{x^2}$$

 $x^2 > 0$  on the range of study. On ]0; 20[, f'(x) < 0 so f is decreasing On [20;  $+\infty$ [,  $f'(x) \ge 0$  so f is increasing. There is minimum for x = 20

4. What is the minimum length of the barrier she has to build?

The minimum is for x = 20 feet so the length of the barrier she has to build is  $f(20) = 2 \times 20 + \frac{400}{20} = 60$  feet.

- → We can notice the enclosure is a square.
- → Possible questions on conversion foot/cm

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris – Créteil – Versailles

Binôme : Anglais / SVT

Corrigé n°D3- n°1

# SEQUENCES

# Exercise 1 :

# Aunt Lucy's Legacy

This morning, Lucy received that letter of her Aunt Lucy.

Dear Lindsay,

Now that I am getting on (I turn 70 today) I want to give you some of my money. I shall give you a sum each year, starting now. You can choose which of the following schemes you would like to use.

- 1. £50 now, £60 next year, £70 the year after and so on.
- 2. £10 now, one and a half as much next year, one and a half as much again the year after and so on.

Of course, the scheme can only operate while I am alive. I look forward to hearing which scheme you choose and why.

Love,

Aunt Lucy

From Website Maths Map, https://www.transum.org/Software/Investigations/Aunt\_Lucy.asp

Help her to do the best choice.

We have to consider each choice separately.

<u>Case 1</u>: it is a sequence in arithmetic progression:

 $\begin{cases} u_0 = 50\\ u_{n+1} = u_n + 10 \end{cases} \text{ so } u_n = 50 + 10n$ So the amount she can hope after n years is  $S1 = n \times \frac{100 + (n-1) \times 10}{2}$ 

<u>Case 2</u>: it is a sequence in geometric progression:

$$\begin{cases} v_0 = 10 \\ v_{n+1} = 1,5v_n \end{cases} \text{ so } v_n = 10 \times 1,5^n$$

So the amount she can hope after n years is  $S2 = 10 \times \frac{1-1,5^n}{1-1,5}$ 

We have to compare the sum of the amount.

n	un	Sum un	vn	Sum vn
0	50	50	10,00	10,00
1	60	110	15,00	25,00
2	70	180	22,50	47,50
3	80	260	33,75	81,25
4	90	350	50,63	131,88
5	100	450	75,94	207,81
6	110	560	113,91	321,72
7	120	680	170,86	492,58
8	130	810	256,29	748,87
9	140	950	384,43	1133,30
10	150	1100	576,65	1709,95
11	160	1260	864,98	2574,93
12	170	1430	1297,46	3872,39
13	180	1610	1946,20	5818,59
14	190	1800	2919,29	8737,88
15	200	2000	4378,94	13116,82
16	210	2210	6568,41	19685,23
17	220	2430	9852,61	29537,84
18	230	2660	14778,92	44316,76

If Aunt Lucy lives less than 9 years, the first option is the best but if she lives 9 years and more, the second will be more interesting. According to the life expectancy in GB, the second choice seems to be the best.