

**BANQUE DE SUJETS** 

# **ANGLAIS / MATHÉMATIQUES**

# SECTION EUROPÉENNE

**SESSION 2023** 

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## Anglais / Mathématiques

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## MERCI DE VEILLER STRICTEMENT À CE QUE LES CANDIDATS RESTITUENT LEUR SUJET UNE FOIS L'INTERROGATION ACHEVÉE.

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## MERCI DE VEILLER STRICTEMENT À CE QUE LES CANDIDATS RESTITUENT LEUR SUJET UNE FOIS L'INTERROGATION ACHEVÉE.

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## MERCI DE VEILLER STRICTEMENT À CE QUE LES CANDIDATS RESTITUENT LEUR SUJET UNE FOIS L'INTERROGATION ACHEVÉE.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet n°1

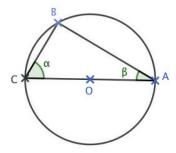
## Thème : Core knowledge

- 1) For each statement, say if it is true or false:
- a) No radii have the same length.
- b) The diameter of a circle is the longest chord.
- c) A circle is all points in the same plane that lie at an equal distance from a given point.
- d) The circumference of a circle is given by the formula  $C = \pi \times D$ , where D is the diameter.
- 2) In English speaking countries, Thales' theorem states that:

"If three points A, B, and C lie on the circumference of a circle, whereby the line AC is the diameter of the circle, then angle  $\angle ABC$  is a right angle."

Let consider the following figure:

Let O be the centre of the circle.



- a) Let consider triangles ABO and BCO: what type of triangles are they? Justify your answer.
- b) What can you say about angles in each triangle?
- c) Without calculating any value of  $\alpha$  nor  $\beta$ , explain how you can prove that the value of  $\alpha$ + $\beta$  is 90°, using the angle sum in triangle ABC.
- d) What type of triangle is triangle ABC?
- 3) Applications:
- a) If ∠BCA = 62°, what is the measure of ∠BAC? And the measure of ∠BOC ? What can you notice?
- **b)** If AB = 6 cm and BC = 8 cm, work out the length of the diameter of its circumscribed circle.

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Binôme : Anglais / Mathématiques

Sujet n°2

#### Domaine 0

## CORE KNOWLEDGE

Yahtzee is a dice game created by Milton Bradley. The objective of the game is to make certain combinations by rolling five dice.

To make a combination, a player gets three rolls of the dice. After the first roll a player may choose not to roll all of the dice. The player may also end their turn before rolling the dice three times.

1. To get the combination called "Large Straight" the player must get five sequential dice (1-2-3-4-5 or 2-3-4-5-6).

Why it is more likely to get a "Large Straight" with 2-2-3-4-5 than with 1-2-2-3-4 at a first roll?

- **2.** After a first roll a player gets 1-2-3-4-4. They decide to roll one of the dice which shows a 4 in order to get a "Large Straight".
  - They may roll this die one or two times.
  - **a.** Draw a tree diagram.
  - **b.** Compute the probability the player gets a "Large Straight".
- **3.** The combination called "Full House" is obtained if the player gets three of one number and two of another.

After a first roll a player gets two threes, two fives and a one.

In order to get a "Full House", they decide to keep the threes and the fives.

- a. What events are leading to a "Full House"?
- **b.** The player may roll the last die once or twice.
  - Let  $W_1$  the event "the player wins at the first roll"
  - Let  $W_2$  the event "the player wins at the second roll"
- Draw a tree diagram to highlight all the possibilities.
- c. What is the probability the player gets a "Full House"?
- 4. (bonus) A "Yahtzee" is obtained if all five dice are the same. At a first roll a player gets three fives, a four and a six. What is the probability to get a "Yahtzee"? Justify.

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Binôme : Anglais / Mathématiques

Sujet n°3

## Thème : D0 – Core knowledge

The first part of this page is a summary that can help you do the exercise.

- 1000 cm<sup>3</sup> = 1L
- If ABE is a right-angled triangle at B:  $\sin \angle ABE = \frac{EB}{AE}$ ;  $\tan \angle ABE = \frac{EB}{AB}$ ;  $\cos \angle ABE = \frac{AB}{AE}$

The following problems are independent and can be solved in any desired order.

**1.** Here are two rectangles.

It is known that : QR = 10 cm, BC = PQ,

AC = 9.6 cm, and the area of PQRS is 45 cm<sup>2</sup>.

Find the perimeter of ABCD, correct to 1 d.p.

**2.** A container is in the shape of a cuboid.

The container is  $\frac{2}{3}$  full of water.

A cup holds 275 ml of water.

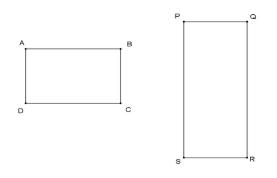
What is the greatest number of cups that can be completely filled with water from the container? How much water will be left?

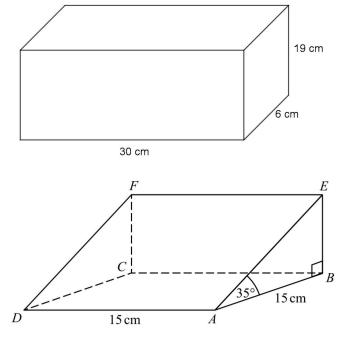
**3.** The diagram shows a triangular prism. The base, ABCD, of the prism is a square of side length 15 cm. Angles ABE and CBE are right angles. Angle EAB =  $35^{\circ}$ . M is the point on DA such that DM : MA = 2 : 3.

a) Check that line-segment MA is 9 cm long.

**b)** Use a trigonometric ratio to find the length of AE in triangle ABE. Give the result to 1 d.p.

**c)** Assuming the fact that triangle AME is rightangled at A, find the value of EM.





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Binôme : Anglais / Mathématiques

Sujet n°4

## Domaine D0 – Core knowledge

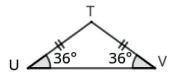
The first part of this page is a summary that can help you do the exercise.

The sum *S* of the interior angles of a *n*-sided polygon is given by the formula :  $S = (n - 2) \times 180$ In a regular polygon, all interior angles have the same measure.

A *golden triangle* is an isosceles triangle in which the ratio of the leg to the base is equal to the Golden Ratio:  $\frac{PQ}{QR} = \varphi$ , where  $\varphi = \frac{1+\sqrt{5}}{2}$ .

We admit that a golden triangle is characterized by its vertex angle:  $\angle QPR = 36^{\circ}$ .

A *silver triangle* is an isosceles triangle in which the base angles are 36° each.

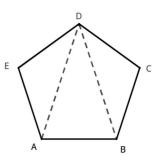


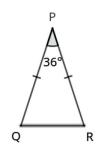
1.

- a) Work out the measure of the base angles in a golden triangle.
- b) Work out the measure of the vertex angle in a silver triangle.
- c) Let PQR be a golden triangle and W the point of segment PQ such that QRW is isosceles.

Prove that QRW is a golden triangle and that WRP is a silver triangle.

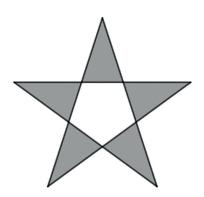
- **2.** Let ABCDE be a regular pentagon.
  - a) Prove that ADE is a silver triangle.
  - **b)** Prove that ABD is a golden triangle.



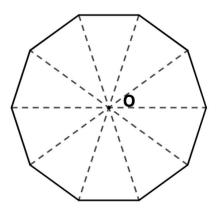


**3.** A regular pentagram is the five-pointed star that is obtained after extending the edges of a regular pentagon.

Justify the shaded triangles (the "branches" of the star) are golden triangles.



**4.** A regular decagon (a 10-sided polygon) with center O can be tiled with ten congruent isosceles triangles having O as the vertex opposite to their base.



Justify that each of these ten triangles are golden triangles.

5. Have you ever heard about the Golden Ratio? If yes, in which context?

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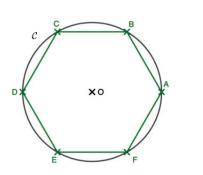
Sujet n°5

Domaine D0

## Archimedes and Pi

Archimedes was a Greek mathematician. He developed a method to find an approximation of Pi  $(\pi)$  using only geometry. The main idea is the following: we can approximate the perimeter of a circle by polygons and deduce an estimation of Pi.

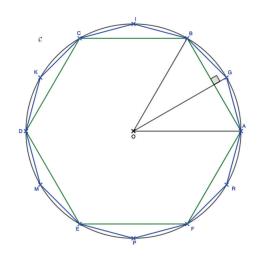
The following figure represents a circle C of radius 1 and center O. Inside the circle, the regular polygon ABCDEF has been drawn.



To find the first approximation of Pi, follow these steps :

- 1. What's the perimeter of this circle ?
- **2.** Knowing that AOB is an equilateral triangle, find the length AB
- **3.** Let p<sub>1</sub> be the perimeter of the ABCDEF. Find p<sub>1</sub> and the first approximation of Pi.

To be able to have a more precise approximation we can double the number of vertices of the polygon. Here is the new figure.



We now would like to find the perimeter  $p_2$  of the polygon AGBICKDMEPFR.

- **4.** We call S the intersection of [OG] and [BA]. Find the length OS. Deduce the length SG (the circle still has a radius of 1).
- **5.** Using the same kind of reasoning, find the value of AG.
- **6.** Find  $p_2$  and give a second approximation of Pi.

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Binôme : Anglais / Mathématiques

Sujet n°6

## Domaine D0 – Core knowledge

When checking out the specification of a smartphone, of a laptop or of a TV, you may have seen the expressions "screen size" and "aspect ratio".

The screen size is the length of the diagonal of the screen. It is often given in inches.

The aspect ratio is the relationship between the height and width of a display. Aspect ratios are commonly written in ratios with the bigger number on the left side and smaller number on the right side. For example, modern TVs normally have an aspect ratio of 16:9, and this means that if the width of the screen is divided into 16 equal parts, the height of the screen should be 9 parts. It is sometimes written as a decimal: for example, a device with a display aspect ratio of 16:9 can also be said to have an aspect ratio of 1.778 (rounded to 3 d.p.), that is to say  $\frac{16}{2}$ .

Common aspect ratios for smartphones are 18:9, 19:9, 20:9, and 21:9.

- **1.** Alice, who has just bought a new laptop, wants to check the screen specifications. She measures its width that turns to be 13.6" and its height 7.65".
  - a) Check that the aspect ratio of the screen is 16:9.
  - **b)** Check that the screen size is 15.6", rounded to 1 d.p.
- **2.** a) Let's suppose a screen is a perfect square. What is its aspect ratio?
  - **b)** Here are two screens A and B: they have the same height, but one has an aspect ratio

of 4:3 while the other one has an aspect ratio of 16:9.

Can you explain which screen has an aspect ratio of 4:3?

screen A	screen B
***************************************	

- c) From a purely mathematical standpoint, what numerical relationship do you recognize between the ratios 4:3 and 16:9?
- **3.** Bob plans to buy a new smartphone with a screen size of 6.5".
  - Let h be the height of the screen and w its width.
  - **a)** Explain why  $h^2 + w^2 = 42.25$ .
  - **b)** Given that h = 2.7", work out *w* rounded to 1 d.p.
- **4.** Would you rather watch a movie on your smartphone, laptop or TV? Why?

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Binôme : Anglais / Mathématiques

Sujet n°7

## The first part of this page is a summary that can help you do the exercise.

There are three methods to solve simultaneous equations.

One of this methods is called the elimination method. You need to follow five steps:

step 1: balance the coefficients of one of the variables ;

step 2: eliminate this variable by adding or subtracting the equations ;

step 3: solve the resulting linear equation in the other variable ;

step 4: substitute the value found back into one of the previous equations ;

step 5: solve the resulting equation.

It takes 4.5 hours for a boat to travel 40.5 miles upstream, the current pushing against the

boat. The same boat can travel 45 miles downstream in 3 hours, thanks to the current pushing the boat faster.

Let B be the speed of the boat and C the speed of the current.

1) Explain why B and C are solutions of the following simultaneous equations:

$$\begin{cases} B - C = 9\\ B + C = 15 \end{cases}$$

If you can't explain the simultaneous equations, admit it and follow up.

2) Solve this simultaneous equations using the elimination method.

**3)** A second method is called the substitution method. Explain it step by step using the previous simultaneous equations.

4) Do you know the third way to solve simultaneous equations?

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Binôme : Anglais / Mathématiques

Sujet n°8

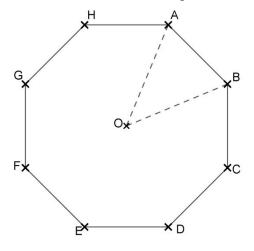
## Domaine 0 – Core Knowledge

## Reminders :

- A regular octagon is a polygon with eight sides of same length. Its vertices can be inscribed in a circle. The centre of this circle is also the centre of the octagon.
- The bearing of B from A is the angle, counted clockwise from the north direction to the direction given by the line segment AB. It is always written with 3 digits. For example,
  - an angle of 65° represents a bearing of 065°;
  - a point C due West from D is on a bearing of 270° from D.

## Exercise (Adapted from Cambridge IGCSE, 2022):

 a) In the regular octagon below, with center O, explain why angle AOB is 45°.
 b) Then deduce the measure of the angle BAO and the measure of angle BAH.



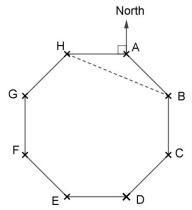
**2.** The diagram shows the route of a boat race.

North

North

B

The route is in the shape of a regular octagon, ABCDEFGH. H is due west of A.



- a) Using information from question 1, find the bearing of B from A.
- **b)** Hence determine the bearing of H from B.

- **3.** Each side of the octagon is 1.35 km. The average speed of a boat is 45 km/h. Work out the time it will take this boat to complete the race. Give your answer in minutes.
- **4.** After the race is finished, a boat is at B and wants to go back to the harbour, which is at H. Work out the distance the boat will sail to reach H from B, correct to the nearest 0.1 km. Explain all your working.
- **5.** One of the contestants wants to draw a scale drawing of the route. He chooses a scale of 1: 500 000.

Has he chosen a suitable scale ? Show all your working and explain your decision.

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Binôme : Anglais / Mathématiques

Sujet n°9

#### Thème: D0 – Core knowledge

The first part of this page is a summary that can help you do the exercise.

Three numbers a, b, c are in the ratio p:q:r where p, q and r are different from 0 if  $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$ 

1) a) Here are simultaneous equations. Solve them, using the method you prefer.

$$\begin{cases} 3x + 2y = 13\\ 2x + y = 8 \end{cases}$$

**b)** Yesterday morning, Joe bought three scones and two muffins at his favourite bakery. He paid £13. His sister, Emma, decided to buy at the same bakery two scones and a muffin to taste them all. She paid £8.

Using the previous question, prove that a scone costs £3 and that the price of a muffin is £2.

**2)** Joe loved the muffins so much that he decided to bake some himself. He found a recipe which uses flour, sugar and butter in the ratio 3:1:1.

**a)** As Joe wants to use 600g of flour, what are the amounts of sugar and butter he needs?

**b)** Given that a muffin weighs 200g, how many muffins can he bake with the previous quantities?

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#### Binôme : Anglais / Mathématiques

#### Sujet n°10

## D0 : CORE KNOWLEDGE

#### Part 1: the bench paradox (probability)

Two men enter a park in which there are three two-place benches. Each man sits randomly. What's the probability of them sitting on the same bench?

- 1) <u>Point of view N° 1</u>: we consider that each man picks a bench at random, no matter which place he sits down on.
  - a) Use a probability tree to determine the universal set of the experiment.
  - b) Let A be the event "The two men are sitting on the same bench".

Find out the outcomes in event A and compute p(A).

- 2) <u>Point of view N° 2</u>: we consider that each man picks a specific place on a bench at random.
  - a) Use a double-entry table to determine the universal set of the experiment.
  - b) Let A be the event "The two men are sitting on the same bench".

Find out the outcomes in event A and compute p(A).

3) <u>Conclusion</u>: explain why there is a paradox here. How do you explain the difference between the two results?

#### Part 2: divisibility criteria (arithmetic)

1) Let n be a three-digit positive integer. There exist three digits a, b anc c such that n = 100a + 10b + c, which can be written  $\overline{abc}$ .

Prove that, if a + b + c is divisible by 3, then so is  $\overline{abc}$  (you can use the fact that 100 = 99 + 1 and 10 = 9 + 1).

- 2) How could we prove that if a + b + c is divisible by 9, then so is  $\overline{abc}$ .
- 3) Do you know of any other divisibility criteria?

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Binôme : Anglais / Mathématiques

Sujet n°11

#### Domaine D0 : Core knowledge

Every question is independent

oer day)
15.00
25.00

**1)** The sign shows the fees charged at a campsite. Today the fees charged were £1260 and there are 18 caravans on the site. How many tents are there?

**2)** In September the total revenue at the campsite was £37 054. This represents a decrease of 4.5% on the total revenue in August. Calculate the total revenue in August.

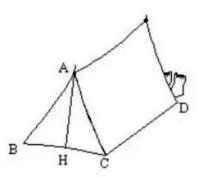
**3)** The visitors to the campsite today are in the ratio men : women = 5 : 4 and women : children = 3 : 7.

(i) Today there are 224 children at the campsite. Calculate the total number of men and women.

(ii) If you pick someone randomly, what is the probability that it is a child?

(iii) Calculate the ratio men : women : children in its simplest form.

**4)** The shape of a tent is a right prism. The height of the triangular shape AH is 2 m and the width BC is 3 m. The length CD is 4 m. What is the volume of the tent?



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Binôme : Anglais / Mathématiques

Sujet n°12

Domaine D0

At a country park there is a house, a museum and a garden. The table shows the prices per person to visit the park.

	Price per person
Garden Only	Free
House and museum	£12.50
House only	£8
Museum only	£7

- 1) One day, 480 people visit the park.
  - 67 visit the garden only.
  - 40% visit the house and the museum.
  - 1/3 visit the house only.
  - The rest visit the museum only.
  - In total, how much have the 480 people paid?

## 2) The next day, there is:

- Twice as many people who visit the "house and museum" than the "garden only"
- 4/3 more people who visit the "House only" than the "house and museum"
- 10 fewer people who visit the "museum only" than the "Garden only".

The revenue of the Park is £ 2 300. How many visitors did the Park have on that day?

**3)** The Park decides to cancel the "House and museum" ticket. It needs £560 per day to be cost-effective<sup>1</sup> by selling "House only" and "Museum Only" tickets.

Let *x* be the number of "House only" tickets to be sold daily and *y* be the number of "Museum Only" tickets to be sold daily.

Express an equation so that the Park can be cost-effective.

<sup>&</sup>lt;sup>1</sup> Cost-effective : rentable.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet n°13

## Thème D0 - Core Knowledge

"Many thousands of riders are now using electric scooters\* daily on US streets shared with millions of pedestrians and drivers," the authors—a group of researchers at University of California, Los Angeles—write.

To catch some of the first injury-related data, the researchers monitored medical records at two emergency departments for a year, skimming for scooter injuries. They nabbed 249 reports of injuries involving electric scooters between September 1, 2017 and August 31, 2018. For comparison, they identified 195 bicycle injuries and 181 pedestrian injuries at the emergency departments during the same time frame.

Of those injured in scooter-related accidents, 58 percent were male and the mean age was 33.7 years old. Around 91 percent of the injured were people riding a scooter at the time of their injury. The remaining were non-riding pedestrians.

The most common type of injuries recorded were head injuries, accounting for 40 percent of scooter injuries overall, the researchers found. Other common injuries included bone fractures (32 percent) and the grouping of contusions, sprains, and lacerations (28 percent).

Researchers noted that only 10 of the injured riders were documented as wearing a helmet despite local laws requiring helmet use.

Adapted from Beth Mole - 1/26/2019 – Ars Technica \* electric scooter = trottinette électrique

**Part A** Using the data provided in this article, answer the following questions, rounding all the results to the nearest unit.

- **1.** What is the ratio between electric scooter injuries and bicycle injuries? Give it in its simplest form.
- **2.** Out of the 249 people that were recorded as being injured due to electric scooters, how many were women?
- **3.** Among the 249 people injured by electric scooters, how many were riding an electric scooter? And how many were non-riding pedestrians?
- **4.** Are there other types of injuries than head injuries, bone fractures and contusions, sprains and lacerations? Explain.
- **5.** With the data given in this article, find the percentage of injured riders who wore no helmet among injured riders.

## Part B

- **6.** Have you ever used an electric scooter? If yes, describe your feelings while riding the scooter. If no, would you like to ride one? Explain why.
- **7.** According to your experience, how is the cohabitation of electric scooters and pedestrians in your home town?

## EPREUVE SPECIFIQUE MENTION « SECTION EUROPEENNE OU DE LANGUE ORIENTALE » Académies de Paris – Créteil – Versailles

## Binôme : Anglais / Mathématiques

Sujet n°1

Domaine 1

Summary

The quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$ $\Delta = b^2 - 4ac$ is called the discrimant of the equation.			
For $\Delta > 0$ , there are two roots: $x_1 = \frac{-b - \sqrt{\Delta}}{2a}$ and $x_2 = \frac{-b + \sqrt{\Delta}}{2a}$			
For $\Delta = 0$ , there is only one root : $x_0 = \frac{-b}{2a}$			
For $\Delta < 0$ , there is no real root.			
Rate: $=\frac{d}{t}$ ; d being distance and t being time.			

Exercise :

Suppose Annette drove 100 km and then increased her speed by 30 km/h for the next 200 miles. The second part of the trip took 1 hour less than the first part of the trip.

We want to determine the average speed for her whole trip.

Let r be the rate for the first part of the trip and t be the time for the first part of the trip.

**1.** Complete the table below:

	Distance in km	Rate in km/h	Time in h
First part of the trip	100	r	t
Second part of the			
trip			
Total	300	XXXXXXX	2t - 1

- 2. a. Express distance in terms of rate and time.
  - **b**. Using the first line of the table, give an equality that r and t satisfy.
  - **c**. Show that: 200 = (r + 30)(t 1).

**d**. Substitute the expression in terms of r for t in the second equation (found in question c).

**e**. Show that the equation you obtained is equivalent to :  $r^2 + 130r - 3000 = 0$  and solve it. Conclude.

**3.** Complete the table with the numeric values

	Distance in km	Rate in km/h	Time in h
First part of the trip	100	20	5
Second part of the	200		
trip			
Total	300	?	

**4.** Calculate the average speed for Annette's whole trip. Does the average speed for the whole trip equal the average of the speeds for the first part and the second part of the trip? Explain.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

## Binôme : Anglais / Mathématiques

Sujet n°2

Domaine 1

Recap :

\* A quadratic function is in the form  $f(x) = ax^2 + bx + c$  with  $a \neq 0$ .

The x-coordinate of the vertex of its graph equals  $-\frac{b}{2a}$ .

To solve  $ax^2 + bx + c = 0$  with  $a \neq 0$ , you calculate the discriminant  $\Delta = b^2 - 4ac$ .

If  $\Delta > 0$ , there are two roots given by the quadratic formula  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2c}$ 

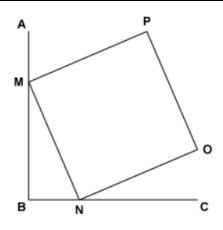
If  $\Delta = 0$ , there is only one root :  $x_0 = \frac{-b}{2a}$ 

If  $\Delta < 0$ , there is no real root.

\* The domain of a function f is the set of all x-values for which f(x) is defined.

The range of a function f is the set of all f(x) values.

\* 1 inch is roughly 2.54 cm.



AB and BC are two perpendicular line-segments so that AB = BC = 5 cm. M  $\in$  [AB] and N  $\in$  [BC] so that AM = BN = x MNOP is a square.

- **1.** Give BM in terms of *x*.
- 2. Find the expression of length MN in terms of *x*. Explain your method.
- **3.** Deduce the expression of the area f(x) of square MNOP.
- **4.** What is the domain of f ? Justify.
- **5.** Describe the graph of the parabola which represents function f.
- 6. What is the minimum area that the square can reach?
- **7.** What is the area of the square when x = 1 cm?
- 8. Determine the range of *f*. Justify.
- 9. a. Solve the equation f(x) = 15.
  b. Express the previous solutions in inches and give the corresponding area in square inches.

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Binôme : Anglais / Mathématiques

Sujet n°3

Domaine 1

#### Recap:

If a quadratic function is  $f(x) = ax^2 + bx + c$  with  $a \neq 0$ , then the x-coordinate of the vertex of its graph is equal to  $-\frac{b}{2a}$ .

To solve  $ax^2 + bx + c = 0$  with  $a \neq 0$ , you can use the quadratic formula  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 

## Exercise:

An activity centre hires out two types of bikes: road bikes and mountain bikes.

The graph shows the cost *C* (in pounds  $\pounds$ ) of hiring a mountain bike for a number of days *d*.

**1)** Among the three given formulae, choose the correct one connecting *C* and *d* for renting a mountain bike.

C = 5d + 15 C = d + 15 C = 15d + 5

Do not forget to explain your choice.

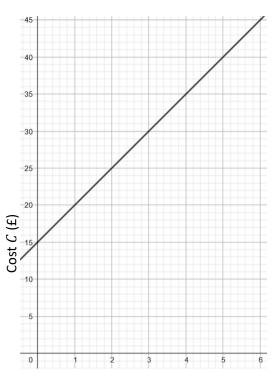
- 2) The cost of renting a road bike is given by the formula C = 10d + 5. Jane wants to rent a road bike since she thinks it will always be cheaper than renting a mountain bike. Is she right?
- **3)** Jane finally decides to rent a mountain bike to climb a small hill next to the activity centre.

Let h be her altitude, in metres, after t hours of riding.

h is given by the formula:

 $h(t) = -800t^2 + 2400t + 500 \; .$ 

- a) What is Jane's altitude at the beginning of her ride?
- b) Can you find the maximum height that Jane will reach?
- c) When will Jane reach an altitude of 1000 meters?



Number of days d

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Binôme : Anglais / Mathématiques

Sujet n°4

Domaine 1

The first part of this page is a summary that can help you do the exercise.

Any guadratic function can be written as:  $f(x) = ax^2 + bx + c$  with a, b and c constants,  $a \neq 0$ . The graph of f is called a parabola. The abscissa of the vertex of the parabola is  $-\frac{b}{2a}$ . To solve a quadratic equation  $ax^2 + bx + c = 0$  with  $a \neq 0$ , calculate the discriminant  $\Delta = b^2 - 4ac$ . • If  $\Delta > 0$ , the quadratic has two real solutions that are:  $x_1 = \frac{-b - \sqrt{\Delta}}{2a}$  and  $x_2 = \frac{-b + \sqrt{\Delta}}{2a}$ • If  $\Delta = 0$ , the equation has one and only one solution which is:  $x_0 = -\frac{b}{2a}$ 

• If  $\Delta < 0$ , the equation has no real solution.

## EXERCISE

A ball is thrown from a rooftop (which is not something you should do) 50m above the ground. The height of the ball is given by the function  $h(t) = at^2 + bt + 50$ , where t represents the time (in seconds) since the ball was thrown and h(t), the height (in meters) of the ball above the ground at this time.

It is given that after one second, the ball is 80m high and after 4 seconds, the ball is 50m high.

- **1.** Write two equations in terms of a or/and b to prove that  $h(t) = -10t^2 + 40t + 50$ .
- 2. When will the ball reach its maximum height? How high will the ball be at that time?
- 3. When will the ball touch the ground?
- **4.** Describe the graph of function *h*.
- 5. When will the ball be higher than 70 meters? Round the answer to 1 decimal place.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet n°5

## Thème : D1-Mapping

The first part of this page is a summary that can help you do the exercise.

Any quadratic function can be written as:  $f(x) = ax^2 + bx + c$  with *a*, *b* and *c* constants,  $a \neq 0$ . The graph of *f* is called a parabola.

The abscissa of the vertex of the parabola is  $-\frac{b}{2a}$ .

To solve a quadratic equation  $ax^2 + bx + c = 0$  with  $a \neq 0$ , calculate the discriminant  $\Delta = b^2 - 4ac$ .

• If  $\Delta > 0$ , the quadratic has two real solutions that are:

$$x_1 = \frac{-b - \sqrt{\Delta}}{2a}$$
 and  $x_2 = \frac{-b + \sqrt{\Delta}}{2a}$ 

• If  $\Delta = 0$ , the equation has one and only solution which is:

$$\kappa_0 = -\frac{b}{2a}$$

• If  $\Delta < 0$ , the equation has no real solution.

Two cyclists, Alice and Bob, move away from a town along two perpendicular paths at 20 m/h and 40 m/h respectively. Bob starts the journey an hour later than Alice.

The objective of this problem is to find how long it would take them to be 100 miles apart.

Let t be the time passed since Alice left, in hours.

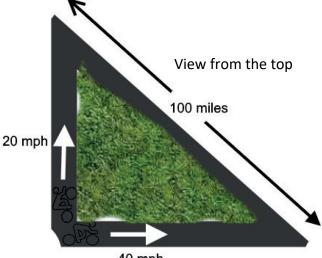
**1.** Express the distance, in meters, travelled by Alice in terms of t.

**2.** Explain why the distance, in meters travelled by Bob is 40(t - 1).

**3.** Hence deduce that the total time taken for the journey satisfies the equation

$$5t^2 - 8t - 21 = 0$$

**4.** Finally find the time taken for them to be 100 miles apart.



#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet n°6

## Thème: D1-Mapping

The first part of this page is a summary that can help you do the exercise.

Any quadratic function can be written as:  $f(x) = ax^2 + bx + c$  with *a*, *b* and *c* constants,  $a \neq 0$ . The graph of *f* is called a parabola.

The abscissa of the vertex of the parabola is  $-\frac{b}{2a}$ .

To solve a quadratic equation  $ax^2 + bx + c = 0$  with  $a \neq 0$ , calculate the discriminant  $\Delta = b^2 - 4ac$ .

• If  $\Delta > 0$ , the quadratic has two real solutions that are:  $x = \frac{-b - \sqrt{\Delta}}{2}$  and  $x = \frac{-b + \sqrt{\Delta}}{2}$ 

$$x_1 = \frac{b}{2a}$$
 and  $x_2 = \frac{b}{2a}$ 

• If  $\Delta = 0$ , the equation has one and only solution which is:

$$x_0 = -\frac{b}{2a}$$

• If  $\Delta < 0$ , the equation has no real solution.

## EXERCISE

During a game, a baseball player throws a ball straight up into the air. The height of the ball is given by the function  $h(t) = at^2 + bt + 6$ , where *t* represents the time (in seconds) since the ball was thrown and *h*, the height (in feet) of the ball above the ground. It is given that after one second, the ball is 86 feet high and after 4 seconds, the ball is 134 feet high.

- **1.** Write two equations in terms of *a* or/and *b* to prove that  $h(t) = -16t^2 + 96t + 6$ .
- 2. When will the ball be at its highest position? How high will the ball be at that time?
- **3.** When will the ball hit the ground? Round to the nearest tenth.
- **4.** Describe the graph of function *h* in a coordinate plane (vertex, x-intercept, y-intercept...).
- 5. When will the ball be higher than 8 feet? Round to the nearest hundredth.
- 6. Can you give another example of real-life application of parabola?

# EPREUVE SPECIFIQUE MENTION « SECTION EUROPEENNE OU DE LANGUE ORIENTALE »

Académies de Paris – Créteil – Versailles

Binôme : langue / DNL

Sujet n°7

## Domaine D1 Mapping HIGH DIVING



Although diving has been a popular pastime across the world since ancient times, the first modern diving competitions were held in England in the 1880s. The exact origins of the sport are unclear, though it likely derives from the act of diving at the start of swimming races. In England, the practice of high diving – diving from a great height – gained popularity; the first diving stages were erected at the Highgate Ponds at a height of 15 feet in 1893 and the first world championship event, the National Graceful Diving Competition, was held there by the Royal Life Saving Society in 1895. The event consisted of standing and running dives from either 15 or 30 feet. Diving is also popular as a non-competitive activity. Outdoor diving typically takes place from cliffs or other rock formations either into fresh or salt water.

 $1 meter = 328,084 \times 10^{-2} feet$ 

You have to talk for ten minutes about this subject. Which mathematical notion(s) do you recognise? The questions may help you, but answering all of them is not compulsory: you can simply explain a way to solve an exercise, even if you can't find the solution

Let us consider a diver named Kyle who is jumping off a cliff.

His trajectory is described according to the following function which assigns the height of the diver (in meters) to the horizontal distance x (also in meters). Its graph is plotted hereunder.

$$f(x) = -0.2x^2 + 0.8x + 15.4$$

- 1) How do you call the shape of this graph ? Compute the actual height of the cliff. How does that compare to the heights found in the competitive practice known as high diving ?
- 2) Show that the function can be written as f(x) = -0.2(x 11)(x + 7)
- 3) Find the distance x for which Kyle enters the water.
- 4) What is the maximal height attained by Kyle during this jump?

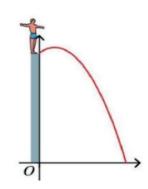
## If you have time :

Base jumpers infer the height of a cliff by measuring the time it takes for a stone thrown down the cliff to be heard. We can approximate the height of such a throw by the formula :

$$h = \frac{1}{2}gt^2$$
 (in meters) with  $g \approx 9.8 \, m. \, s^2$  and the duration *t* (in seconds)

5) Determine the height of a cliff which yields an echo after a time of t = 4 seconds.

The speed of sound is roughly  $340 \text{ m. s}^{-1}$ 



#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet n°8

## Thème : D1 - MAPPING

The first part of this page is a summary that can be helpful to do the exercise.

The standard form equation of a quadratic function is:  $f(x) = ax^2 + bx + c$  with a, b and c constants,  $a \neq 0$ . The graph of f is called a parabola.

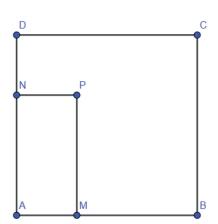
DO NOT WRITE ON THE SUBJECT!!

The abscissa of the vertex of the parabola is  $-\frac{b}{2a}$ 

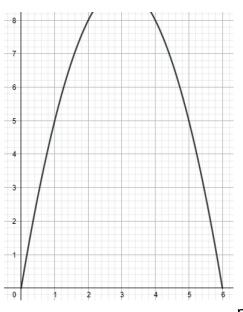
## EXERCISE

Consider a square ABCD. Let M be a point of [AB] and N a point of [AD] such as:

- AB = 6 cm
- AM = DN = x
- AMPN is a rectangle.
- 1) What are the possible values of *x*?



2) Determine the area of the rectangle AMPN in terms of *x*.



Here is an incomplete graph of the function f representing the area of AMPN against x.

**3)** Describe the graph of the function f (name, shape, x-intercept, y-intercept, vertex...).

**4)** Find the value of the area of AMPN when AM is equal to 5 cm.

**5)** For what distances AM will the area be greater than 6 cm<sup>2</sup>?

**6)** Where should M be so that the area is as large as possible?

**7)** Can you give another example of real-life application of parabola?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet n°9

## Domaine D1

The first part of this page is a summary that can help you do the exercise.

- A quadratic function is a polynomial in the form  $f(x)=ax^2+bx+c$ , where a, b, c are real numbers. Its curve is a parabola which has a vertex for  $x = -\frac{b}{2a}$ .
- To solve a quadratic equation ax<sup>2</sup> + bx + c = 0 with a ≠ 0, one may use the discriminant Δ = b<sup>2</sup> 4ac.

If  $\Delta > 0$ , the quadratic has two real solutions that are:  $x_1 = \frac{-b - \sqrt{\Delta}}{2a}$  and  $x_2 = \frac{-b + \sqrt{\Delta}}{2a}$ 

If  $\Delta = 0$ , the equation has one and only solution which is:  $x_0 = -\frac{b}{2a}$ 

If  $\Delta < 0$ , the equation has no real solution.

Given the equation of a line L: y=ax+b, any perpendicular line to L has a slope (or gradient) of -<sup>1</sup>/<sub>a</sub>.

## Exercise.

Given that  $f(x) = x^2 - 4x + 5$ , define over all real numbers.

1. Express f(x) in the form  $(x + a)^2 + b$ 

where a and b are integers to be found.

- Let A be the y-intercept of the curve y=f(x).
   Let B be the minimum turning point of y=f(x)
  - **a.** Write down the coordinates of A
  - **b.** Give the coordinates of B
  - c. Compute the coordinate of M, the midpoint of the segment-line AB.
- 3. Lines.
  - **a.** Find the equation of the line (AB).
  - **b.** Find the equation of *p*, the perpendicular line to AB that passes through M
  - **c.** What does *p* represent for the segment line AB?
- **4.** Find the intersections, if they exist, of the line p and the curve y=f(x).

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

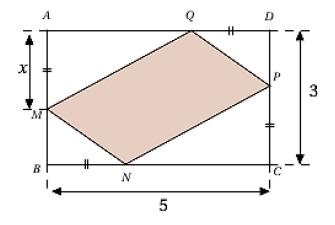
Binôme : Anglais / Mathématiques

Sujet n°10

D1

Let *ABCD* be a rectangle such as AB = 3 cm and AD = 5 cm. Points *M*, *N*, *P* and *Q* belong to sides *AB*, *BC*, *CD* and *AD* such as *AM* = *BN* = *CP* = *DQ* = *x*.

Let A(x) be the area of the quadrilateral *ABCD* depending on *x*.



- 1) What kind of quadrilateral is MNPQ? You don't need to justify.
- 2) What is the maximum area of *MNPQ*? Explain your reasoning.
- 3) a) What is the area of the right triangle AMQ?
  - **b)** What is the area of the right triangle MRN?
  - **c)** Deduce that  $A(x) = 2x^2 8x + 15$ .
- 4) Can we place *M* such as *MNPQ* has an area of 9 cm<sup>2</sup>?
- 5) What is the minimum area of MNPQ? What is the area then?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet n°11

Domaine 1

## Reminders :

Any quadratic function can be written as:  $f(x) = ax^2 + bx + c$ .

Its curve is a parabola which has a vertex for  $x = -\frac{b}{2a}$ .

The discriminant of a quadratic trinomial is the expression  $b^2 - 4ac$ .

- If this expression is negative, the trinomial has no real root.
- If this expression equals zero, the trinomial has a double root:  $x = -\frac{b}{2a}$ .
- If this expression is positive, the trinomial has two different real roots:  $x = \frac{-b \pm \sqrt{b^2 4ac}}{2}$

## Exercise :

To celebrate the end of the year, fireworks are launched in a park from a platform 29 feet off the ground. The height, en feet, of the fireworks can be modeled as a function of time *t*, in seconds, using the function  $h(t) = at^2 + bt + c$ , where *a*, *b* and *c* are real numbers.

It is also known that after 1 second, the fireworks are at an altitude of 125 feet, and after 2 seconds, its altitude is 189 feet.

- **1.** Using the information above, explain why c = 29.
- **2.** Explain why the data can be expressed as  $\begin{cases} a+b=96\\ 2a+b=80 \end{cases}$
- **3.** Solve the system above and show that  $h(t) = -16t^2 + 112t + 29$ .
- **4.** Describe the graph of the function h(t).
- **5.** Suppose one of the fireworks does not go off\*\* as intended. Where does it land back on the ground ? Explain your work.
- 6. Determine the maximum altitude a firework can reach.
- **7.** The fireworks must go off at least at 200 feet from the ground, to ensure the safety of the onlookers. In what time-interval could the fireworks go off? Explain your work.

#### \*\*go off = explode

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Binôme : Anglais / Mathématiques

Sujet n°12

## MAPPING / D1

The first part of this page is a summary which will help you solve the following exercise.

Any quadratic function can be written as:  $f(x) = ax^2 + bx + c$ . Its curve is a parabola which has a vertex for  $x = \frac{-b}{2a}$ . Any quadratic trinomial can be written as:  $ax^2 + bx + c$ . Its discriminant is the expression  $b^2 - 4ac$ . If this expression is negative, the trinomial has no real root. If this expression equals zero, the trinomial has a double root:  $x = \frac{-b}{2a}$ . If this expression is positive, the trinomial has two different real roots:  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 

## Exercise

A businessman has just moved into Mathville, a city situated in California. He wants to build an amusement park on the large plain that stretches just outside of town. He needs a permit from the Mayor.

"What surface area do you need?", asks the Mayor.

"About 200 square miles", answers the businessman.

"All right", says the Mayor, "You may choose a rectangular piece of land. Its dimensions must be such that if the width of the rectangle were 11 miles longer and the length 9 miles longer, the area of the rectangle would be three times greater. Besides, its perimeter must be equal to 58 miles".

The businessman duly selected his land in accordance with these conditions. But he got away with eight square miles more than what the Mayor had anticipated.

- 1) Make a short presentation of the text.
- 2) Let W be the width of the rectangle and L be its length. Prove that solving the problem is the same as solving the following system :  $\begin{cases} W + L = 29\\ 2WL = 11L + 9W + 99 \end{cases}$
- 3) Show that solving this system relies on solving the equation  $2W^2 60W + 418 = 0$ .
- 4) What was the value the Mayor had in mind?
- **5)** A second businessman wants to build a zoo near the city on a piece of land shaped as a square whose surface area is equal to 4 mi<sup>2</sup>.
  - **a.** Compute the perimeter of the square.
  - **b.** Knowing that ten miles of fencing cost half a million dollars, how much will it cost to build a fence around the park?
- c. The businessman is offered a 15 % discount. How much will he pay then for the fence?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet n°13

## Thème: D1 – Mapping

The first part of this page is a summary that can help you do the exercise.

Any quadratic function can be written as:  $f(x) = ax^2 + bx + c$  with a, b and c constants,  $a \neq 0$ . The graph of *f* is called a parabola.

The abscissa of the vertex of the parabola is  $-\frac{b}{2a}$ .

To solve a quadratic equation  $ax^2 + bx + c = 0$  with  $a \neq 0$ , calculate the discriminant  $\Delta = b^2 - 4ac.$ 

- If  $\Delta > 0$ , the quadratic has two real solutions that are:  $x_1 = rac{-b - \sqrt{\Delta}}{2a}$  and  $x_2 = rac{-b + \sqrt{\Delta}}{2a}$
- If  $\Delta = 0$ , the equation has one and only solution which is:

$$x_0 = -\frac{b}{2a}$$

• If  $\Delta < 0$ , the equation has no real solution.

A magic square is a square in which the sum of numbers of each row, each column and both main diagonals are the same.

The aim is to complete the following magic square by replacing each letter by the corresponding number.

Here are some clues to find the values of the letters *a*, *b*, *c*, *d*, *e*, *f*.

1) *a* is the number which satisfies the equality:

$$x^2 - 2x - 2 = (x - a)^2 - 3.$$

**2) b** is the product of the two roots of the quadratic function  $2x^2 + 11x - 6$ .

**3)** *c* is the number of solutions of the equation  $x^2 + 2x + 3 = 0$ .

4) *d* is the minimum of the quadratic function  $x^2 + 18x + 85$ .

5) *e* is the y-coordinate of the intersection point of the linear functions y = x - 3 and y = -3x + 21.

**6)** *f* is the greatest preimage of 2 for the function  $f(x) = x^2 + 6x + 10$ .

7) Now, you can complete the magic square. Be ready to explain all your results!

8) Do you know other Maths games? Can you explain the benefits of those games?

a		е
	с	f
b	d	

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet n°1

## Differentiation

The first part of this page is a summary that can be helpful to do the exercise.

Let f be a function defined and differentiable over an interval I

If f'(x) > 0 over *I*, then *f* is an increasing function over *I*.

If f'(x) < 0 in over *I*, then *f* is a decreasing function over *I*.

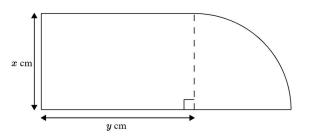
A stationary point  $a \in I$  of the graph of f is a point where the gradient f'(a) is equal to 0.

It is a maximum if f''(a) < 0 and a minimum if f''(a) > 0.

Let C be a circle of radius r.

- The perimeter of the circle is given by the formula  $2\pi r$ .
- The area of the circle is given by the formula  $\pi r^2$

## EXERCISE



The diagram above shows a metal plate consisting of an x cm by y cm rectangle and a quartercircle. The perimeter of the plate is 60 cm.

1) What is the radius of the circle?

**2)** Express y in terms of x.

**3)** Let  $f(x) = 30x - x^2$ .

- **a)** Give the range where f(x) > 0
- **b)** find the stationary values of f(x).
- c) Is it a maximum or a minimum value?
- d) What are the dimensions of the plate in this case?

Adapted from Cambridge AS&A Level Mathematics

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

#### Binôme : Anglais / Mathématiques

Sujet n°2

## DIFFERENTIATION

The first part of this page is a summary which may help you solve the following exercise.

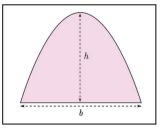
Applications of derivatives

Let f be a function defined and differentiable over an interval I

- If f'(x) > 0 over *l*, then *f* is an increasing function over *l*.
- If f'(x) < 0 in over *I*, then *f* is a decreasing function over *I*.

A stationary point  $a \in I$  of the graph of f is a point where the gradient f'(a) is equal to 0. It is a maximum if f''(a) < 0 and a minimum if f''(a) > 0.

The area of a parabolic segment is found by the formula  $\frac{2}{3}b \times h$  where b is the segment's base length and h, the height of the parabola.



#### Exercise :

During an epidemic, the number of patients in thousands , *t* days after the occurrence of the first cases, is modelled by a quadratic function  $f(t) = at^2 + bt + c$ 

We know that after 10 days after the occurrence of the first cases, there are 120,000 patients, and 20 days after the occurrence of the first cases, there are 210,000 patients. After 90 days the epidemic has been eradicated.

**Part A:** We want to determine *a*, *b* and *c* 

- **1.** Write a linear system of 3 equations in a, b and c using the information given above
- 2. Solve the linear system

**Part B:** Let  $f(t) = -0.15t^2 + 13.5t$ 

- 1. Give the number of patients after 30 days.
- 2. Give the maximum number of patients during this epidemic.
- 3. The hospital is under pressure if the number of patients is at least 270,000. For how many days is the hospital going to be under pressure?
- To evaluate the average number of patients during this epidemic, we have to compute the area under the parabola and divide the result by the length of the epidemic.

Give the average number of patients during this epidemic per day.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet n°3

## Domaine : Differentiation

The first part of this page is a summary that can be helpful to do the exercise.

## **APPLICATIONS OF DERIVATIVES**

Let f be a function defined and differentiable over an interval I.

If f'(x) > 0 over *I*, then *f* is an increasing function over *I*.

If f'(x) < 0 over *I*, then *f* is a decreasing function over *I*.

On a graph, f'(a) is the gradient of the tangent to the curve of f at the point (a, f(a)).

A stationary point  $a \in I$  of the graph of f is a point of the graph where f'(a) is equal to 0.

It is a maximum if f''(a) < 0 and a minimum if f''(a) > 0.

#### Exercise

Blood Alcohol Content (BAC) is a measure of the amount of alcohol present in a certain amount of blood. It is usually described as the amount of alcohol in grams per litre of blood. The maximum prescribed legal drink driving limit in England and Wales is 0.8g/l and in Scotland it is 0.5g/l<sup>1</sup>.

We've studied the BAC in someone's blood (expressed in grams per litre of blood) twice within the five hours following an alcohol ingestion, first on an empty stomach,

then after eating.

**1)** Here are two curves, one showing the results when the person had an empty stomach, and the other when the person had eaten.

a) Identify each curve (explain your choice).

**b)** In each case, give an approximate value of the maximum BAC. When is it reached?

**c)** In each case, check if the person can drive in the UK after 3h. And in Scotland?

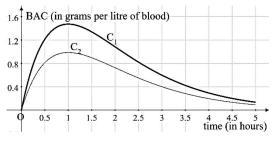
d) If f is the function shown by the graph, f' is the velocity of alcohol presence variation in blood. When is the maximum velocity reached in each case?

**2)** Eliott is a young Scottish driver. Tonight, he's invited to a party in Edinburgh, and he'll use his car to go. He would like to drink two pints of beer. His mother is a biologist and has established that, with the absorption of these two beers, the concentration of alcohol in Eliott's blood is given by the formula:  $f(t) = 2te^{-t}$  where *t* is the time, in hours, after drinking.

**a)** Check that, for any positive value of t,  $f'(t) = 2e^{-t}(1-t)$ .

b) What is the maximum BAC for Eliott in this situation? When is it reached?

c) Eliott needs to know when he will be authorised to drive again. Using your calculator, give the answer to the nearest ten minutes. Explain your reasoning.



<sup>&</sup>lt;sup>1</sup> Remember that in France the legal drink driving limit is 0.2 g/l for young drivers.

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Binôme : Anglais / Mathématiques

Sujet n°4

#### D2

## DIFFERENTIATION

The first part of this page is a summary that can be helpful to do the exercise.

Let f be a function defined and differentiable over an interval I

If f'(x) > 0 over *I*, then *f* is an increasing function over *I*.

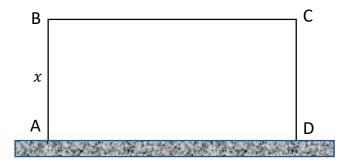
If f'(x) < 0 in over *I*, then *f* is a decreasing function over *I*.

A stationary point  $a \in I$  of the graph of f is a point where the gradient f'(a) is equal to 0.

It is a maximum if f''(a) < 0 and a minimum if f''(a) > 0.

## Exercise

Emmy wants to build a rectangle enclosure for her animal (a goat) with a surface of 800 square feet. In order to minimize the costs, she plans to build it against her house and wonders which minimum length of barrier she has to buy to surround the enclosure. She draws the figure below.



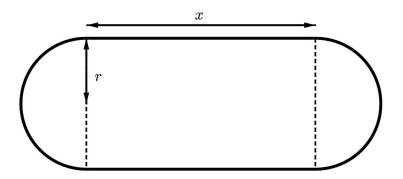
- **1.** Let *x* be the length *AB* and *y* be the length *BC*. Express *y* in terms of *x*.
- **2.** Express the perimeter in term of *x*
- **3.** Let *f* be the function defined by  $f(x) = 2x + \frac{800}{x}$ .
  - **a.** What is the range of *f*?
  - **b.** Give the variations of f.
  - **c.** Find the stationary point of the function *f*, and solve Emmy's problem.

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Binôme : Anglais / Mathématiques

Sujet n°5

Domain 2



The figure above shows the design of a stadium track and field track. The track consists of two semicircles, each of radius r metres, joined up to a rectangular section of length x metres. The total length of the track is 400 metres. The area  $\mathcal{A}(r)$  depends on the value of r.

- **1)** Using the total length of the track, show that  $x = 200 \pi r$ .
- **2)** Deduce that the expression of the area is  $\mathcal{A}(r) = 400r \pi r^2$ .

In order to hold field events safely, it is required for the area enclosed by the track to be as large as possible.

- 3) Explain why there is a value of r at which the area  $\mathcal{A}(r)$  reaches a maximum and then calculate it.
- 4) Show that the maximum area enclosed by the track is  $\frac{40\ 000}{\pi}\ m^2$ .
- 5) Explain why the resulting shape of the track may not be suitable.
- 6) Do you know any other method to find the maximum value of the area ?
- 7) Using the total length of the track, show that  $x = 200 \pi r$ .
- 8) Deduce that the expression of the area is  $\mathcal{A}(r) = 400r \pi r^2$ .

In order to hold field events safely, it is required for the area enclosed by the track to be as large as possible.

- 9) Explain why there is a value of r at which the area  $\mathcal{A}(r)$  reaches a maximum and then calculate it.
- **10)** Show that the maximum area enclosed by the track is  $\frac{40\ 000}{\pi}\ m^2$ .
- **11)** Explain why the resulting shape of the track may not be suitable.
- 12) Do you know any other method to find the maximum value of the area?

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

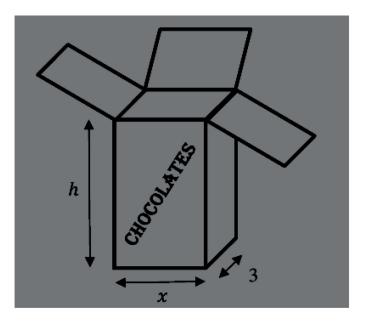
Sujet n°6

# **Domaine D2 – Differentiation**

A box of chocolates has a rectangular basis and is closed thanks to 3 identical rectangles.

The width of the basis is 3 inches.

Let x be the length and h the height of the box, in inches, as shown below.



- 1) Give the volume of the box in terms of h and x.
- 2) The volume has to be 180 cubic inches in order to contain the right number of chocolates.Give *h* in terms of *x*.
- 3) Let *A* be the area of cardboard necessary to build this box.

Show that:  $A(x) = 12x + 120 + \frac{360}{x}$ , for x > 0.

- 4) Work out the derivative of *A*.
- 5) Draw the variation table of *A*.
- **6)** What is the minimum area of the cardboard box, in square inches? And in square centimeters (one inch equals 2.54 cm)?

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Binôme : Anglais / Mathématiques

Sujet n°7

Domain 2: Differentiation



Nearly a third of Britons believe insects will eventually be part of mainstream human diets in the UK amid mounting challenges in food production, new research reveals.

Fried crispy silkworms on sale at a street food stall in Thailand. Photograph: Engdao Wichitpunya/Alamy

With UK farmers facing pressure from the climate crisis, pests and plant diseases – alongside the need to boost productivity and compete with imports – research released on Monday claims that 32% of British adults think that regularly tucking into cricket snacks and buffalo worm burgers will become commonplace within 10 years.

The research from the Agricultural Biotechnology Council (ABC), an industry group promoting the use of controversial genetically modified (GM) crops in the UK, also suggests that 72% of people support increased emphasis on technology, such as new plant breeding techniques including gene editing, to tackle crop shortages.

Insects are also nutritious, containing essential proteins, fats, minerals and amino acids. Bugs for consumption are typically bred in large-scale factory conditions.

The global edible insect market is set to exceed \$520m (£430m) by 2023, according to recent research. The UN Food and Agriculture Organisation says at least 2 billion people regularly consume insects. But while more than 1,000 species are eaten around the world, they hardly feature in the diets of many rich nations.

Extracted from: <u>https://www.theguardian.com/food/2019/sep/02/grubs-up-a-third-of-britons-think-well-be-</u> eating-insects-by-2029

- A. Explain what the text deals with and comment on it.
- **B.** In his genetics laboratory, Daniel is studying the variations in numbers of a population of insects in a closed enclosure for a month: he puts some insects, food and water into an aquarium and seals it for 31 days.

He notices that the number of insects follows the quadratic function  $f(x) = -\frac{x^2}{2} + 20x + 25$ , in which x represents the day of the month.

- 1. How many insects are present at the beginning of the study?
- 2. How many insects will there be on the last day of the month?
- **3.** The derivative of the function f, denoted by f', is the evolution speed of the insects' population.
  - **a.** Compute the derivative of f and study its signs on interval [0,31].
  - **b.** When was the evolution speed the highest? The lowest?
- **4. a.** Using the sign of f', draw the table of variations of f.

b. What day was the population at its highest point?

**c.** Describe the evolution of the insects' population over the month. How can you interpret this evolution?

### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet n°8

# Thème **DIFFERENTIATION**

The first part of this page is a summary that can be helpful to do the exercise.

# Applications of derivatives

Let *f* be a function defined and differentiable over an interval *I* If f'(x) > 0 over *I*, then *f* is an increasing function over *I*. If f'(x) < 0 in over *I*, then *f* is a decreasing function over *I*.

A stationary point  $a \in I$  of the graph of f is a point where the gradient f'(a) is equal to 0.

It is a maximum if f''(a) < 0 and a minimum if f''(a) > 0.

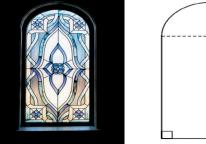
Let C be a circle of radius r.

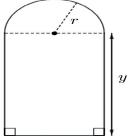
- The perimeter of the circle is given by the formula  $2\pi r$ .
- The area of the circle is given by the formula  $\pi r^2$

# EXERCICE

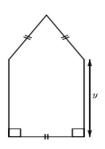
Many objects, such as windows, are composite shapes. The stained-glass window represented is made up of a rectangle and a semi-circle.

A window company is interested in exploring the relationship between the perimeter of the window and its area. It also wants to look at maximizing the area for a fixed perimeter.





- 1. The perimeter of the window is fixed at *P* metre.
  - **a.** Find an expression for the height y of the rectangular section in terms of the radius r.
  - **b.** Compute the area,  $A m^2$ , of the window in the form  $A = ar^2 + br$  where *a* and *b* are constants.
  - c. the dimensions of the window that maximize its area and the maximum area for *P*=6*m*
- 2. Investigate a second stained-glass window, as represented, with an equilateral triangle top section. Compare its maximum area with question 1, for the same perimeter of 6 m.



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Binôme : Anglais / Mathématiques

Sujet n°1

# D3 / Sequences

The first part of this page is a summary that can be helpful to do the exercise.

# Arithmetic sequence

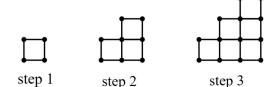
A sequence  $(a_n)$  is an arithmetic progression (AP) with common difference *d* if it can be written  $a_{n+1} = a_n + d$ , where  $n \ge 1$ .

The *n*-th term of an arithmetic sequence whose first term is  $a_1$  and common difference *d* can be written  $a_n = a_1 + (n-1)d$ .

For any integer n,  $1 + 2 + \dots + n = \frac{n(n+1)}{2}$  and  $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$ 

# EXERCISE

Max uses matches to make the following diagrams, always following the same pattern.



**1)** Draw the diagram corresponding to the next step (step 4), following the same pattern. Let  $u_n$  be the number of matches used for step n.

**2)** Could  $u_n$  be an AP or a GP? Explain your answer.

3) Complete the following table:

Step n	1	2	3	4
$u_n =$ number of matches	4	10		
$n^2$				
$v_n = u_n - n^2$				

**4)** Assume that  $(v_n)$  follows the same pattern

a) What sort of sequence is it exactly?

**b)** Express  $v_n$  in terms of n.

**5) a)** Express  $u_n$  in terms of n.

**b)** How many matches will Max need at step 10?

c) There are 240 matches in a box of matches. How many boxes will Max need to complete the first 10 steps? Could he make more steps with these boxes?

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Binôme : Anglais / Mathématiques

Sujet n°2

# Thème : D3 – Suites – Sequences

### Document 1 : Victoria Amazonica

*Victoria Amazonica* is the largest of all water lilies<sup>(1)</sup> with huge circular leaves that, incredibly, are strong enough to support the weight of a child. (...) The seeds are the size of peas.(...) They germinate well when placed in warmer water and develop over the next few weeks. (...) By this time, the *Victoria* has leaves about the size of a CD (...) The leaves are at their biggest in June. At this time of the year, the *Victoria Amazonica* will produce a fully grown leaf in about a week, from a small bud to a leaf potentially over 2 m wide in just seven days.

Source: S.Taylor, 11/05/2020, https://www.kew.org/read-and-watch/growing-propagation-giantwaterliles

### Questions

Considering a water lily at the beginning of its growth, let  $u_n$  be the diameter (in centimetres) of the water lily after n days.

- **1.** Given that  $u_0 = 12$  and that the diameter of the water lily increases by 50 % every day, work out  $u_1$  and  $u_2$ .
- **2.** Write  $u_{n+1}$  in terms of  $u_n$ .
- **3.** Is  $(u_n)$  an arithmetic progression? A geometric progression?
- **4.** Express  $u_n$  in terms of n.
- **5.** a) What is the diameter of the water lily after 1 week? (to 3 s.f.)

b) At this time, what is the area<sup>(2)</sup> of the water lily? (to 2 s.f.)

**6.** If the growth continued, how long would it theoretically take for the water lily to cover a 10 meter wide pond<sup>(3)</sup>?

(1) Water lily: a plant whose large, flat leaves and petals float on the surface of lakes and pools.

- (2) area of a disc:  $\pi R^2$
- (3) pond: an area of water smaller than a lake, often artificially made.

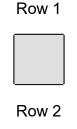
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Binôme : Anglais / Mathématiques

Sujet n°3

# Domain 3 Sequences :

Anna has some sticks that are all of the same length. She arranges them in squares and has made the following 3 rows of patterns:









She notices that 4 sticks are required to make a single square in the first row, 7 sticks to make 2 squares in the second row and 10 sticks to make 3 squares in the third row.

- **1.** How many sticks does Anna need to make the  $6^{th}$  row.
- **2.** Find an expression, in terms of n, for the number of sticks required to make a similar arrangement of n squares in the  $n^{th}$  row.

Anna continues to make squares following the same pattern. She makes 4 squares in the  $4^{th}$  row and so on until she has completed 10 rows.

**3.** Find the total number of sticks Anna needs to make 10 rows.

Let k be the number of rows Anna can complete starting with 1750 sticks.

- Show that k satisfies 3k<sup>2</sup> + 5k 3500 ≤ 0.
   (*Hint : Write the formula which gives the total number of sticks needed to complete k rows in terms of k*).
- **5.** Find the value of k.

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Binôme : Anglais / Mathématiques

Sujet n°4

# Domaine : Sequences

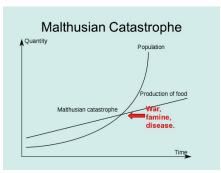
The first part of this page is a summary that can be helpful to do the exercise.

A sequence  $(a_n)$  is an arithmetic sequence with common difference d if it can be written  $a_{n+1} = a_n + d$ , where  $n \ge 0$ . The *n*-th term of an arithmetic sequence whose first term is  $a_0$  and common difference d can be written  $a_n = a_0 + nd$ .

A sequence  $(b_n)$  is a geometric sequence with ratio r if it can be written  $b_{n+1} = b_n \times r$ , where  $n \ge 0$ . The n-th term of a geometric sequence with first term  $b_0$  and common ratio r can be written  $b_n = b_0 \times r^n$ .

# Exercise

In 1798, a 32-year-old British economist anonymously published a lengthy pamphlet criticizing the views of the Utopians who believed that life could and would definitely improve for humans on earth. The hastily written text, *An Essay on the Principle of Population as it Affects the Future Improvement of Society, with Remarks on the Speculations of Mr. Godwin, M. Condorcet, and Other Writers*, was published by Thomas Robert Malthus.



Thomas Malthus's example of population growth doubling was based on the preceding 25 years of the brand-new United States of America.

Malthus felt that a young country with fertile soil like the U.S. would have one of the highest birth rates around. He liberally estimated an arithmetic increase in agricultural production.

He wrote: 'Population, when unchecked, goes on doubling itself every 25 years or increases in a geometrical ratio' and 'the product of the land might be increased every 25 years, by a quantity equal to what it at present produces'.

**1)** The total population in England in 1800 was 10 million. We assume that agriculture could feed everyone at that time.

**a)** Prove that, following Malthus theory, doubling the population of England every 25 years corresponds to an annual growth of 2.8%

**b)** Explain why after 25 years the product of the land could feed the double of the population and then show that the annual increase is 0.4 million.

**2)** For any whole number  $n \ge 0$ , in the year 1800 + n, let's note  $p_n$  the total population in England (in million) and  $a_n$  the number of people (in million) who can be fed by English agriculture, still following Malthus theory. We have  $p_0 = a_0 = 10$ .

**a)** Show that  $p_n$  is a geometric sequence and  $a_n$  an arithmetic sequence. Give all the information you have about these two sequences.

**b)** Following this model, will the product of the land be enough to feed everyone in England in 1820? And in 1830?

c) Using your calculator, find the year when the product of the land becomes insufficient

3) a) Could explain the graphs given at the beginning of the exercise?

- b) What is the Malthusian catastrophe? Did it happen?
- c) Do you think this theory is still working nowadays?

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Binôme : Anglais / Mathématiques

Sujet n°5

# **SEQUENCES : Domaine D3**

The first part of this page is a summary that can help you do the exercise.

### Arithmetic sequence

A sequence  $(u_n)$  is an arithmetic progression (AP) with common difference d if it can be written  $u_{n+1} = u_n + d$ , where  $n \ge 1$ .

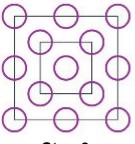
The *n*-th term of an arithmetic sequence whose first term is  $a_1$  and common difference *d* can be written:  $a_n = a_1 + (n-1)d$ .

**EXERCISE**: Aleya uses coins to make the following diagrams, maintaining the same pattern.





Step 2





- 1) Explain how to draw the diagram corresponding to step 4.
- 2) How many coins does she need to add:
  - from step 1 to step 2?
  - from step 2 to step 3?
  - from step 3 to step 4?

Let *n* be a natural number and  $c_n$  be the number of coins used for step *n*.

- **3)** Explain why Aleya can write  $c_{n+1} = c_n + 4n$ . What is the value of  $c_1$ ?
- 4) Let  $(u_n)$  be the sequence defined as  $u_n = c_n 2n^2$  for  $n \ge 1$ .
  - **a)** Show that  $(u_n)$  is an arithmetic sequence with common difference d = -2.
  - **b)** Express  $u_n$  in terms of n.
  - c) Deduce that  $c_n$  can be expressed in terms of n by  $c_n = 2n^2 2n + 1$ .
- 5) How many coins will Aleya need at step 15?

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# Binôme : Anglais / Mathématiques

Sujet n°6

Domaine D3

An influencer on a social media is trying to anticipate the evolution of their number of followers. Indeed, they want to attend an event on the 1<sup>st</sup> of March 2024 but needs at least 40 000 followers to be allowed in. On the 1<sup>st</sup> of January 2023, they had 4 000 followers. By studying the evolution of the previous year, they figure out that at the beginning of every month their number of followers doubled but afterwards they lost 1000 followers.



To study the evolution, they modeled their number of followers by a sequence  $(u_n)$  with  $u_n$  the number of followers, in thousands, n-month after January 2023 and  $u_0 = 4$ .

- **1.** Explain that we can write  $u_{n+1} = 2u_n 1$ .
- 2. Show that this sequence is neither arithmetic nor geometric.
- **3.** To be able to their evolution, they decide to introduce another sequence:  $v_n = u_n 1$ .
  - a. What kind of sequence is (vn)?
  - **b.** Give the general form of  $(v_n)$  and deduce the explicit form of  $(u_n)$ .
- **4.** Using your calculator, give the number of followers this influencer will have on the 1<sup>st</sup> of March 2024 according to this model. Can they attend the event ?
- 5. After how many years they have more than 500 K followers ?

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Binôme : Anglais / Mathématiques

Sujet n°7

# Domain 3: Sequences



Trapped in a metal cage in a corner of a 350,000 sq ft Amazon warehouse outside Boston last week a lonely yellow robot arm sorted through packages, preparing items to be shipped out to customers who demand ever-faster delivery. Soon it will be joined by others in a development that could mean the end of thousands of jobs the creation of thousands of others

and, Amazon argues, the creation of thousands of others.

One day soon the robot, called Sparrow, could do the work of the hundreds of thousands of people that Amazon now employs to sort the 13m packages it delivers each year. Using computer vision and artificial intelligence Amazon says Sparrow can already identify about 65% of its product inventory, tell if an item is damaged and discard it, and adjust its suction cup "hand" to handle different objects – all jobs currently done by human hands. As it learns, it gets better by the day.

Extracted from: <a href="https://www.theguardian.com/technology/2022/nov/11/amazon-robots-jobs">https://www.theguardian.com/technology/2022/nov/11/amazon-robots-jobs</a>

- A. Explain what the text deals with and comment on it.
- **B.** Marianne has recently started a new job in a factory in which she has to package goods.

The first day, as a beginner, she was able to package only 15 boxes. The second day, she packaged 18 boxes. As she gets more comfortable and faster, every day she packages 3 more boxes than the day before.

Let  $u_1$  be the number of packaged boxes the first day,  $u_2$  the number of packaged boxes the second day and so on.

- **1.** a. What are the values of  $u_1$  and  $u_2$ ?
  - b. Write  $u_{n+1}$  in terms of  $u_n$ . What kind of sequence is it?
  - c. Write  $u_n$  in terms of n.
- 2. How many boxes is Marianne able to package after one week of working at the factory?
- 3. The goal is to package a hundred boxes a day. How long does she need to reach this goal?
- **C.** Matthias also started the same job. The first day, he packaged only 10 boxes. Then every day, he packaged 10% more than the day before.

Let  $v_1$  be the number of packaged boxes the first day,  $v_2$  the number of packaged boxes the second day and so on. In the following questions, round the results to the nearest whole number if needed.

- a. What are the values of v<sub>1</sub> and v<sub>2</sub>?
   b. Write v<sub>n+1</sub> in terms of v<sub>n</sub>. What kind of sequence is it?
   c. Write v<sub>n</sub> in terms of n.
- 2. How many boxes is Matthias able to package after one week of working at the factory?
- 3. How long does he need to reach the goal of 100 packages a day?
- D. How long does it take for Matthias to pack faster than Marianne?

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

Sujet n°8

Domaine D3

#### Arithmetic sequence

A sequence  $(u_n)$  is an arithmetic progression (AP) with common difference d if it can be written

 $u_{n+1} = u_n + d$ , where  $n \ge 1$ .

The *n*-th term of an arithmetic sequence whose first term is  $a_1$  and common difference *d* can be written:  $a_n = a_1 + (n-1)d$ .

The sum of the first *n* terms of an AP is given by  $S_n = a_1 + \dots + a_n = \frac{n(a_1 + a_n)}{2}$ .

#### The Lazy Caterer's Sequence

Suppose you have a large pizza, or a pancake, if you prefer. You are allowed to cut it using straight lines only, and the resulting pieces are not necessarily the same size.

If you cut your pizza once, you get two pieces, if you cut it twice, you get four pieces.

The problem starts when you cut it the third time. What is the maximum number of pieces you can get ?

Note that you can increase the number of pieces if you avoid the intersection point of two cuts.

Let  $u_n$  be the **maximum** number of pieces you can get when cutting your pizza (or pancake) with n cuts, where n is a natural number.

You should use drawings throughout the exercise.

- 1) Explain why  $u_3 = 7$  and  $u_4 = 11$ .
- 2) The sequence v of « first differences » is formed by the differences of the terms of  $u_{...}$

The sequence w of « second differences » is formed by the differences of the terms of v.

Copy and complete the table below with the first and the second differences. The gray cells are not to be completed.

n	0	1	2	3	4	5
u <sub>n</sub>	1	2	4	7	11	16
First differences $v_n = u_n - u_{n-1}$						
$(n \ge 1)$						
Second differences w <sub>n</sub>						
S <sub>n</sub>						

- 3) What do the second differences tell you about the nature of  $v_n$ ?
- 4) Show that for all  $n \ge 1$ , we can add at most n extra pieces at the n-th step of the process.
- **5)** Let  $S_n$  be the sum of the first n + 1 terms of the sequence v.

$$S_n = v_0 + \dots + v_n$$

Give the formula of  $S_n$  and fill in the last row of the table above.

**6)** Deduce the formula of  $u_n$ .

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Binôme : Anglais / Mathématiques

Sujet n°9

# **D3: SEQUENCES**

# Aunt Lucy's Legacy

The first part of this page can be useful to do the exercise.

The sum of the first $n$ terms of an arithmetic progression with first term $a$ and common ratio $r$ is:
$n \times \frac{2a + (n-1)r}{2}$
$n \times \frac{2}{2}$
The sum of the first $n$ terms of a geometric progression with first term $a$ and common ratio $r$ is:
$a \times \frac{1-r^n}{1-r}$
$u \times \frac{1-r}{1-r}$

This morning, Lindsay received that letter of her Aunt Lucy.

Dear Lindsay,

Now that I am getting old (I turn 70 today) I want to give you some of my money. I shall give you a sum each year, starting now. You can choose which of the following options you would like to use.

- 1. £50 now, £60 next year, £70 the year after and so on.
- 2. £10 now, one and a half as much next year, one and a half as much again the year after and so on.

Of course, the option can only operate while I am alive. I look forward to hearing which option you choose and why.

Love,

Aunt Lucy

From Website Maths Map, https://www.transum.org/Software/Investigations/Aunt\_Lucy.asp

Help her make the best choice.

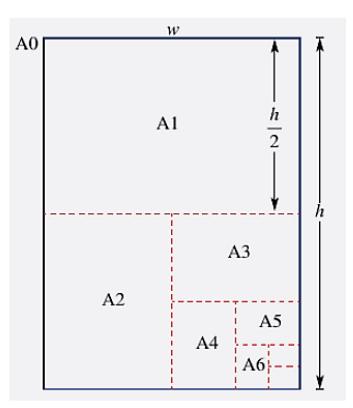
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Binôme : Anglais / Mathématiques

Sujet n°10

# Thème SEQUENCE

The A series of paper sizes, e.g. A4, are based on international standards. If we split an A0 paper into two equal parts we get two A1 papers ; and if we split an A1 paper into two equal parts we get two A2 papers ; and so on. See the picture below.



The paper sizes are such that the ratio between the height and width of each paper size is the same.

The height is taken to be the longer side length of each rectangle.

Let an A0 piece of paper have width w mm and height h mm.

For every integer  $n \ge 0$ , let  $w_n$  and  $h_n$  be the width and height (in mm) of a An piece of paper. In particular, we have  $w_0 = w$  and  $h_0 = h$ . 1. Complete the table below for the corresponding height and width of the A paper series in terms of w and h.

An	Width <i>w</i> <sub>n</sub>	Height <i>h<sub>n</sub></i>
A0	W	h
A1	$\frac{h}{2}$	W
A2		
A2 A3		
A4		
A5		
A6		

- **2.** Remember that, if we split an An paper into two equal parts along the long side, we get an An + 1 paper: write  $w_{n+1}$  in terms of  $h_n$  and  $h_{n+1}$  in terms of  $w_n$ .
- **3.** The ratio  $\frac{h_n}{w_n}$  has to be the same for all paper sizes, in particular for A0 and A1 paper sizes. Determine the value of the ratio  $\frac{h}{w}$ .
- **4.** From your results in question 3, write a relation for the height  $h_n$  in terms of the width  $w_n$ , for all integer  $n \ge 0$ .
- **5.** From your results in questions 2 and 4, write a recursive formula for the sequences  $(w_n)$  and  $(h_n)$ .
- **6.** A0 paper has an area of 1 square meter. Determine the dimensions, *w* and *h*, of A0 paper in exact form in mm.
- **7.** Use your results to determine the length and width in mm for A4 paper. Do those values look familiar to you?

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## Binôme : Anglais / Mathématiques

Sujet n°11

Domaine D3

# Exercise 1

Diane is using a device to extract air from a bottle of wine. This helps preserve the wine left in the bottle. She has to perform several extractions in order to extract as much air as possible. The more she pumps, the less air there is in the bottle.

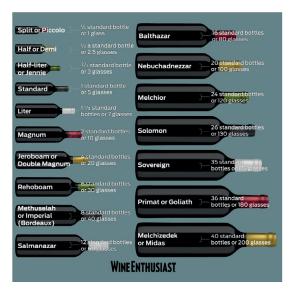
Diane's first attempt extracts 45 cm<sup>3</sup> and her second attempt extracts 36 cm<sup>3</sup>. The volume of the subsequent extractions follows a geometric sequence.

- 1. Give the common ratio of this sequence.
- 2. Calculate the third term of the sequence.
- **3.** Express the n-th term of the sequence in terms of n.
- **4.** Calculate the sum of the first ten terms of this sequence. How can we interpret this sum given the context?

Exercise	2

Bottle name	Bottle capacity				
Small	0,20 Liters				
Quarter	0,25 Liters				
Half or Girl	0,375 Liters				
Medium	0,50 Liters				
Bottle	0,75 Liters				
Magnum	1,5 Liters				
Jeroboam	3 Liters				
Rehoboam	4,5 Liters				
Methuselah	6 Liters				
Salmanazar	9 Liters				
Balthazar	12 Liters				
Nebuchadnezzar	15 Liters				
Melchior or Salomon	18 Liters				
Sovereign	26,25 Liters				
Primate	27 Liters				
Midas or Melchiesedech	30 Liters				

- Do the bottle capacity values follow a usual (geometric or arithmetic) sequence?
- If not, can some of them be put together so that they follow a geometric or an arithmetic sequence?



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#### Binôme : Anglais / Mathématiques

Sujet n°1

# STATISTICS / D4

The first part of this page is a summary that will help you to do the exercise.

A **box and whisker** plot or diagram (otherwise known as a boxplot), is a graph summarizing a set of data.

The **median** is a measure of central tendency which divides a set of observed values into two equally sized groups.

In statistics, a **quartile** is a type of quantile which divides the number of data points into four parts, or *quarters*, of more-or-less equal size.

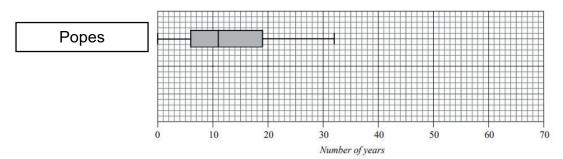
#### **EXERCISE**

The length of reign of each of the last 19 monarchs is given in the table.

George VI	16 years	George IV	10 years	James II	3 years
Edward VIII	0 years	George III	60 years	Charles II	25 years
George V	26 years	George II	33 years	Charles I	24 years
EdwardVII	9 years	George I	13 years	James I	22 years
Victoria	64 years	Anne	12 years	Elizabeth I	45 years
William IV	7 years	William III	14 years	Mary	5 years
				Edward VI	6 years

1) Order the data set from the lowest to the greatest value. (You can represent the data in an ordered stem and leaf diagram).

- 2) Find the median and quartiles of the length of reign of these 19 monarchs. You **must** show calculations to support your answer.
- 3) Calculate the range and the interquartile range.
- 4) Write down the name of any monarch whose length of reign is an outlier.
- 5) The box and whisker plot shows the length of reign of the last 19 popes.



Draw a box and whisker plot for the length of reign of the last 19 monarchs on a copy of the diagram. **6**)

- a. Statement 1: 25 % of the reigns of the last 19 popes are greater than 19 years
- b. Statement 2: 50 % of the reigns of the last 19 popes are less than 11 years

c. Statement 3: 20 % of the reigns of the last 19 monarchs are greater than 20 years

Are the statements true or false? Explain your reasoning.

7) Compare the length of reign of the monarchs and popes.

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Binôme : Anglais / Mathématiques

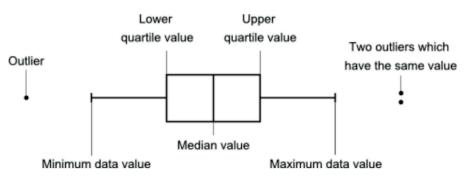
Sujet n°2

# STATISTICS / D4

The first part of this page is a summary that will help you do the exercise.

A **box and whisker** plot or diagram (otherwise known as a boxplot), is a graph summarising a set of data. The shape of the boxplot shows how the data is distributed and it also shows any <u>outliers</u>.

In statistics, an outlier is a data point that differs significantly from other observations.



The following quantities (called *fences*) are needed to identify extreme values in the tails of the distribution:

1.lower inner fence: Q1 - 1.5\*IQ

2.upper inner fence: Q3 + 1.5\*IQ

A point beyond an inner fence on either side is considered a **mild outlier**.

### EXERCISE

In the film « Moneyball », Beane (Brad Pitt) and assistant general manager Peter Brand (Jonah Hill), faced with the franchise's limited budget for players, build a team of undervalued talent by taking a sophisticated sabermetric approach to scouting and analyzing players. Data science and statistics are fundamental and decisive.

The table on the back lists the total number of homeruns<sup>1</sup> hit at away games by each team in Major League Baseball (MLB) during the 2010 season (sorting in descending order). The scores are ordered from the greatest to the lowest.

<sup>&</sup>lt;sup>1</sup> A homerun is a hit that allows the <u>batter</u> to make a complete circuit of the bases and score a run.

Home Runs in 2010 MLB Games Played Away								
Team	Homerun	Team	Homerun	Team	Homerun			
Boston	113	Milwaukee	82	Atlanta	65			
Toronto	111	Arizona	82	NY Mets	65			
Minnesota	90	Tampa Bay	82	Colorado	65			
San Francisco	87	Chicago Cubs	75	Cleveland	64			
LA Angels	86	Washington	75	Oakland	63			
NY Yankees	86	San Diego	73	Pittsburgh	62			
Cincinnati	86	Philadelphia	72	Baltimore	61			
St Louis	83	Texas	69	Kansas City	61			
Florida	83	Chicago Sox	66	LA Dodgers	59			
Detroit	82	Seattle	66	Houston	45			

- **1.** Find the median and quartiles of the score of homeruns of the 30 teams. You **must** show calculations to support your answer.
- 2. Calculate the range and check that the interquartile range (IQR) is 18.
- A box plot displays a five number summary of a data set. The five numbers are: minimum, low quartile, median, third quartile, maximum. Draw a box and whisker plot for the score of homeruns of the 30 teams. Unit: 1cm for 10 homeruns.
- 4. Compute the fences and determine whether the data set includes any outliers.
- 5. Are the statements true or false? Explain your reasoning.
  - **a.** Statement 1: Roughly 25 % of the homeruns of the 30 teams are greater than 83.
  - **b.** Statement 2: Roughly 50 % of the homeruns of the 30 teams are less than 74.
  - **c.** Statement 3: No more than 20 % of the homeruns of the 30 teams are greater than 75.
- 6. Why do you think data science is so important in Sports?

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

### Binôme : Anglais / Mathématiques

Sujet n°3

# **<u>Thème</u>** : Domaine 4 – Statistiques

The first part of this page is a summary that can be helpful to do the exercise.

Given a statistical ordered distribution:

- The Median is a value that split the ordered distribution in two halves
- The lower quartile (LQ) is a value one-quarter on the way into the distribution
- The upper quartile (UQ) is a value three-quarter on the way into the distribution.

### **EXERCISE.** Women in politics

The following table gives the percentage of women in parliaments from 23 of the most populated countries in the world.

Country	Percentage	Russia	16	Germany	31
China	25	Japan	10	Turkey	15
India	12	Mexico	42	France	39
USA	19	Philippines	29	UK	32
Indonesia	20	Ethiopia	39	Thailand	5
Brazil	11	Vietnam	27	Italy	35
Pakistan	22	Egypt	15	Source:	
Nigeria	6	Congo	11	http://archive.	.ipu.org/wmn
Bangladesh	20	Iran	6	<u>-f/classif.htm</u>	

1) Find the median of this sample. Comment the value you found.

2) Give the lower and upper quartile. What do these values show?

3) Compute the interquartile range. Comment your result.

4) Represent these data in a boxplot.

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## Binôme : Anglais / Mathématiques

Sujet n°4

# Statistics

The first part of this page is a summary that can be helpful to do the exercise. Given a statistical ordered distribution:

- The Median is a value that split the ordered distribution in two halves
- The lower quartile (LQ) is a value one-quarter on the way into the distribution
- The upper quartile (UQ) is a value three-quarter on the way into the distribution.

# EXERCISE

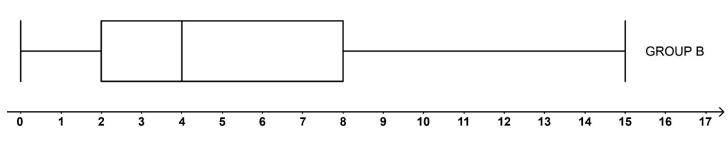
A survey has been conducted on a group (group A) of people who caught COVID-19 even though they were vaccinated. They were asked how many days they had symptoms. The results are registered in the following table:

Number of days with symptoms	0	1	2	3	4	5	6	7	8
Frequency	17	8	11	6	8	10	4	5	1

#### **Group A**

- 1) How many people took part in this survey?
- 2) Give the percentage of people who had no symptoms. Round the percentage to 1 d.p.
- 3) a. Explain how you can find the median of this data (no calculation is asked here).
  - **b**. The median is 2. Explain what it means in context.
- 4) Prove that the interquartile range is 4.
- 5) a. Sketch the box-plot for group A below (above the boxplot of group B)

**GROUP** A



**b.** The other box-plot (group B) drawn above corresponds to another group of people who caught COVID-19 but who were not vaccinated.

By comparing both groups, discuss the vaccine's efficacy.

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#### Binôme : Anglais / Mathématiques

Sujet n°5

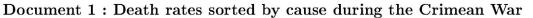
# D4 Statistics: Florence Nightingale's Rose Charts

Florence Nightingale is best remembered for her work as a nurse during the Crimean War (1853-1856) and her contribution towards the reform of the sanitary conditions in military field hospitals. However, what is less well known about this amazing woman is her love of mathematics, especially statistics, and how this love played an important part in her life's work.

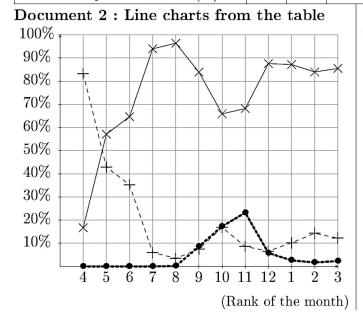
Although being female meant Nightingale had to fight against the military authorities at every step, she went about reforming the hospital system. Back then, injured soldiers were 7 times more likely to die from disease in hospital, than on the battlefield.

Using her statistics, she illustrated the need for sanitary reform in all military hospitals. While pressing her case, Nightingale gained the attention of Queen Victoria and Prince Albert as well as that of the Prime Minister, Lord Palmerston. Her wishes for a formal investigation were granted in May 1857 and led to the establishment of the Royal Commission on the Health of the Army. In 1858, for her contributions to army and hospital statistics Nightingale became the first woman to be elected to be a Fellow of the Royal Statistical Society.

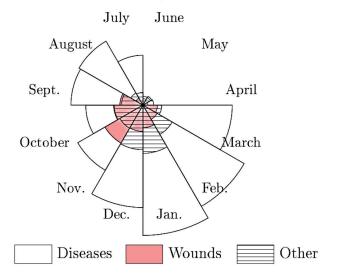
Adapted from MacTutor History of Mathematics Archive



	v			0			3.12 3.11 8.823					
Month $(1854-1855)$	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.
Death by avoidable diseases $(\%)$	16.7	57.1	64.7	94.0	96.4	83.9	65.9	68.2	87.6	87.2	84.0	85.5
Death by wounds $(\%)$	0.0	0.0	0.0	0.0	0.1	8.6	17.3	23.2	5.8	2.6	1.7	2.3
Death by other causes $(\%)$	83.3	42.9	35.3	6.0	3.5	7.5	16.8	8.6	6.6	10.2	14.3	12.2



Document 3 : Florence Nightingale's Rose Chart from the same period



**1.** Match each line (plain line, dashed line, dotted line) in Document 2 with its corresponding death cause from Document 1. Explain how you proceed.

**2.** In Document 3, each represented area is made to be proportional to the number of dead soldiers. According to the rose chart, what is the main cause of death? Is that surprising given the context?

3. On which document can you see what the deadliest month is?

**4.** If you had to convince people that sanitary changes are needed, which chart would you choose and why?

**5.** Let's consider, in Document 1, the percentage of death by avoidable diseases during the Crimean War.

- a) Calculate the mean and the median.
- b) Interpret in the context of the exercise.

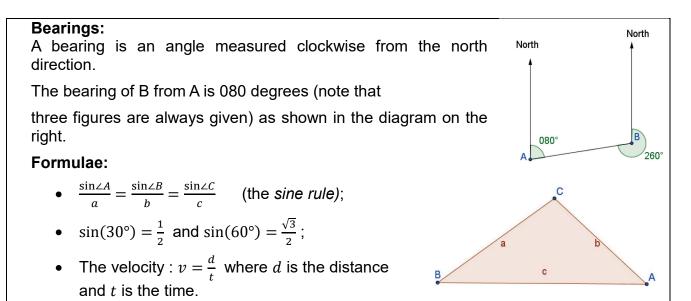
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Binôme : Anglais / Mathématiques

Sujet n°1

# Thème: D5 - Advanced Geometry

The first part of this page is a summary which will help you solve the following exercise



# EXERCISE:

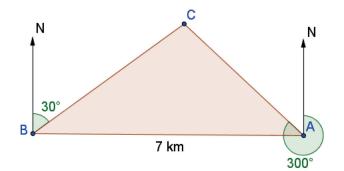
Two fire lookout towers are located in a national park close to a lake. They are 7 kilometres away from each other.

The map on the right shows the towers viewed from the sky.

Points *A* and *B* represent the towers. The arrows point northwards.

At point *C*, a fire starts on a bearing of  $030^{\circ}$  from *B* and on a bearing of  $300^{\circ}$  from *A*.

- **1)** Work out the measures of angles  $\angle A$ ,  $\angle B$  and  $\angle C$  in the *ABC* triangle.
- 2) What can you say about triangle ABC?
- **3)** Work out the bearing of Tower *B* from Tower *A*.
- 4) Calculate the distance between Tower *B* and the fire using the sine rule.
- 5) Calculate the distance between *A* and *C*. Round to 2 dp.
- 6) A fire-fighting plane can fly from tower *A* to the fire at a speed of 303 km. h<sup>-1</sup>.
  A fire truck can go from tower *B* to the fire at a speed of 70 km. h<sup>-1</sup>.
  Which vehicle will reach the fire first?



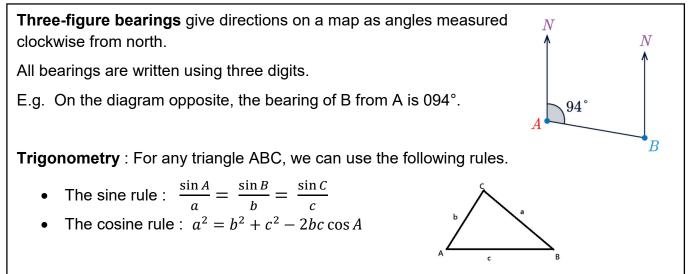
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Sujet n°2

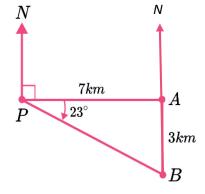
# Domaine D5 : Advanced geometry

The first part of this page is a summary which will help you solve the following exercise.



# Exercise:

- **1)** On this diagram, calculate :
  - a) the bearing of B from P
  - b) the bearing of P from B
  - c) the length PB rounded to the nearest whole number.



- **2)** Starting from a point A, a ship sails at a bearing of 053° for 80 km and reaches point B. From B it then sails at a bearing of 165° for 135 km until it reaches point C.
  - a) Draw a sketch of the ship's trajectory.
  - b) What is the bearing of B from C?
  - c) Find the distance AC, round the result to 1 dp.
  - **d)** What bearing should the ship sail on to get back to A from C? Round the answer to the nearest whole number.

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »

Académies de Paris – Créteil – Versailles

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# PROBABILITY

D7 – Sujet n°1

The first part of this page is a summary that can help you do the exercise.

For any events *A* and *B* of the sample space:  $P(A) = P(A \cap B) + P(A \cap \overline{B})$   $P(\overline{B}) = 1 - P(B)$ 

# Conditional probability:

Let *A* and *B* be two events. The probability of *A* given *B* is  $P(A|B) = \frac{P(A \cap B)}{P(B)}$  provided that  $P(B) \neq 0$ .

Binomial distribution:

If the random variable X gives the number of successes in *n* trials, then X follows the binomial distribution with parameters n and p.

The expectation of a random variable X following a binomial distribution of parameters n and p is equal to np.

# EXERCISE

Elsa plans to plant two types of flowers in her garden: 30% are tulips and the rest are roses. Both types will yield either white or yellow flowers.

If the flower is a tulip, the probability that it is white is 0.4. If not, the probability that the flower is yellow is 0.2.

Let *T* be the event "the flower is a tulip", R the event "the flower is a rose", W the event "the flower is white" and *Y* the event "the flower is yellow".

- 1. Draw a tree diagram.
- 2. Calculate the probability that a flower, chosen at random, will be white.
- 3. Suppose a white flower is randomly picked from Elsa's garden, what is the probability of the flower being a tulip?
- 4. Anna, Elsa's sister, randomly picked 10 flowers from her garden. Suppose that Elsa has planted a large amount of flowers, we can consider that Anna picked 10 flowers with replacement. Let *X* be the random variable counting the number of white flowers picked by Anna.
  - a) Explain why *X* follows a binomial distribution and give its parameters.
  - b) Find the probability that Anna picked at least five white flowers (round to the 3<sup>rd</sup> d.p.).
  - c) Calculate the expectation of *X* . Interpret in the context of the exercise.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

#### PROBABILITY

D7 – Sujet n°2

For any events *A* and *B* of the sample space:

 $P(A) = P(A \cap B) + P(A \cap \overline{B})$  $P(\overline{B}) = 1 - P(B)$ 

# **Conditional probability:**

Let *A* and *B* be two events. The probability of *A* given *B* is  $P(A|B) = \frac{P(A \cap B)}{P(B)}$  provided that  $P(B) \neq 0$ .

### **Binomial distribution:**

If the random variable X gives the number of successes in *n* trials, then X follows the binomial distribution with parameters n and p.

The expectation of a random variable *X* following a binomial distribution of parameters n and p is equal to np.

# EXERCISE

John likes playing tennis, and today he has a match to play in Cambridge.

1) When John has to serve, he is allowed two serves.

The probability that his first serve is in is 0.8.

He has probability 0.7 of winning the point when his first serve is in. Otherwise, his probability of winning the point is only 0.4.

- a) Draw the tree diagram to represent this information.
- b) Find the probability that he wins the point when he is serving.
- c) Given that he does win the point, find the probability that his first serve was out.
- 2) Let assume that usually, John's probability of winning a point (serving or receiving) is 0.57.

Today, during the first set, we can assume that all the points are independent since John is not tired yet. During this first set, 60 points are played.

Let *X* be the random variable counting the number of points won by John.

- a) Explain why *X* follows a binomial distribution and give its parameters.
- b) What is the probability that John wins 40 of them?
- c) Calculate the expectation of X. Interpret in the context of the exercise.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

## PROBABILITY

D7 – Sujet n°3

Julia is an independent consultant who has to travel from Paris to London on the Eurostar for a special work assignment. She will earn £800 as a basis salary plus a bonus of £100 if she manages to catch the 8 a.m. train or a bonus of £200 if she manages to catch the 7 a.m. train.

Julia is waiting for a train at the Gare du Nord in Paris and is placed on standby: she is not guaranteed a definite seat. If there is no seat on a given train, then she waits to see if there is a seat on the next train.

This information is provided for Julia at the station.

Train	Probability of a seat	Cost
7 a.m.	$\frac{1}{2}$	£ 290
8 a.m.	$\frac{2}{3}$	£ 200
9 a.m.	$\frac{3}{4}$	£ 150
No further train	-	£0

# Part A

- 1. Using a tree diagram, show all Julia's options and the probabilities for the three trains (*all branch paths might not have the same length*).
- 2. a) Prove that the probability of catching the 9 a.m. train is  $\frac{1}{a}$ .
  - b) Find the probability that Julia will catch
    - the 7 a.m. train
    - the 8 a.m. train.
- 3. What is the probability that Julia will miss all available trains?

# Part B

4. Tabulate the cost of travel, the total salary (including the potential bonus) and the profit (salary minus cost) of each outcome and its corresponding probability.

<u> </u>	/		1 51	,
Train	7 a.m.	8 a.m.	9 a.m.	No further train
Cost (£)				£0
Total Salary (£)				£ 0
Profit (£)				£ 0
Probability				

5. Find Julia's expected (average) profit for that day.

#### ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris-Créteil-Versailles

Binôme : Anglais / Mathématiques

#### PROBABILITY

D7 – Sujet n°4

The first part of this page is a summary which may help you solve the following exercise.

Let A and B be two events:

- $A \cap B$  is the event "both A and B occur"
- the probability that A occurs, given that B has occurred, is denoted by  $P_B$  (*A*)and, provided that  $P(B) \neq 0$ ,  $P_B$  (*A*) =  $\frac{P(A \cap B)}{P(B)}$

#### Exercise:

For screening tests, the aim is to detect as many people as possible who could be infected with the SARS-CoV-2 coronavirus in order to isolate them and prevent them from infecting others. We want to ensure that infected people are identified. What we want to maximize, in this case, is the sensitivity, i.e., the proportion of true positives among all infected individuals. This is the probability that a test is positive for an infected person: the higher the probability, the more sensitive the test is.

For diagnostic tests, the aim is to establish a diagnosis, i.e., to find out whether a person is infected with the SARS-CoV-2 coronavirus. It is then important to make sure that the disease is detected. We want to make sure that people who are not infected are correctly identified. What you want to maximize in this case is the specificity, i.e., the proportion of true negatives among all uninfected individuals. This is the probability that a test will be negative for a person who is not infected: the higher the probability, the more specific the test is.

To evaluate a test, both its specificity and sensitivity must be considered. The ideal is to choose a test that optimizes both sensitivity and specificity.

From the website "adios Corona"

In France, the "Haute Autorité de la Santé" considers that diagnostic tests for COVID-19 must have a minimum specificity of 99% and a minimum sensitivity of 80%.

We want to evaluate if a test meets the "Haute Autorité de la Santé" criteria

For this test, 58% of a sample of patients are diagnosed positive. 98% of the patients diagnosed positive are infected, and 89% of the patients diagnosed negative are not infected.

Let T be the event: "the test is positive" and I be the event "the patient is infected".

- 1. Draw a tree diagram representing the situation.
- 2. a. Calculate the probability that a patient is diagnosed positive and is infected.
  - b. Calculate the probability that a patient is diagnosed negative and is infected.
  - c. Calculate the probability that a patient is infected.
- 3. Evaluate the specificity and the sensitivity.
- 4. Conclude.

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »

# Académies de Paris – Créteil – Versailles

#### Binôme : Anglais / Mathématiques

D7- sujet n°5

#### Summary

**Conditional probability**: A and B are two events.  $A \cap B$  is the event "both A and B occur"

Law of total probability :  $P(B) = P(A \cap B) + P(B \cap \overline{A}) = P(A) \times P_A(B) + P(\overline{A}) \times P_{\overline{A}}(B)$ .

The probability that A occurs, given that B has occurred, is  $P_B(A) = \frac{P(A \cap B)}{P(B)}$  with  $P(B) \neq 0$ ,

▶ **Binomial distribution**: Suppose that a trial is repeated *n* times, and that in each trial there are two possible outcomes *A* and  $\overline{A}$ . If P(A) = a and the trials are independent from each other,  $P(A \text{ occuring } k \text{ times}) = \binom{n}{k} a^k (1-a)^{n-k}$  where  $\binom{n}{k}$  is the number of paths leading to *k* successes in *n* experiments.

• When X follows a binomial distribution B(n, p), the **expected value of X** is  $E(X) = n \times p$ 

#### Exercise :

In times of pandemic, there is a lot of talk about the importance of testing. However, it is tempting to take the result of a test for granted: if I test positive, I am infected; if I test negative, I am not.

Unfortunately, tests are not 100% reliable: you may test positive and not be infected – this is called a *false positive*; you may test negative and nevertheless be sick – this is called a *false negative*.

In order to understand what it means to receive a positive or negative result from a test, we need two pieces of information:

- The prevalence of the disease, i.e. the proportion of the population who is infected;
- The *accuracy* of the test, i.e. the probability that the test gives a correct result.

The accuracy of a test is normally measured with two numbers: *sensitivity* and *specificity*.

Sensitivity is the proportion of true positives among all infected individuals.

Specificity is the proportion of true negatives among all uninfected individuals.

It is tempting to think that if the test is accurate 95% of the time, if you receive a positive result, you have a 95% chance of being infected (and conversely, if you receive a negative result, you have a 95% chance of being healthy). THIS IS NOT THE CASE!

Different sources, mostly a blog Michel Baudouin -Lafon

- 1. If the prevalence is 10 %, sensitivity 95% and specificity 95% as well, calculate the probability to be infected given that your test is positive (rounded to 2 d.p.).
- 2. a. Let x be the prevalence of the disease, sensitivity is 95% and specificity 95% as well, calculate the probability of being infected given that your test is positive in terms of x.
  - b. Find the value for x so that the probability to be infected given that your test is positive is at least 95%.

3. Let's assume that the probability to be tested positive is 0.14.

We suppose that in a class, there are 35 students, and we assume that the outcomes of each of them being tested positive on a chosen day are independent from each other.

Let *X* be the random variable that counts the number of students tested positive.

- a. Show that *X* follows a binomial law and give its parameters.
- b. What is the probability for at least a fifth of the students to be tested positive on a chosen day?
- c. On average, how many students can be tested positive in the class on a chosen day?

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »

Académies de Paris - Créteil - Versailles

Binôme : langue / DNL

D 7- sujet n°6

<u>Recap</u>: Let A and B two events:

\*  $A \cap B$  is the event "both A and B occur"

\* the probability that A occurs, given that B has occurred, is denoted by P(A|B) and, provided that  $P(B) \neq 0$ ,  $P(A|B) = \frac{P(A \cap B)}{P(B)}$ 

\* A and B are independent if  $P(A \cap B) = P(A) \times P(B)$ 

\* Imagine n identical and independent trials that are repeated with a probability p of success for each trial. If the random variable X gives the number of successes in n trials, then X follows the binomial distribution with parameters n and p

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

The expectation of the binomial distribution is given by  $E(X) = n \times p$ .

Exercise:

Jim owns a snack bar in Bath. In his bar, in May, he offers two types of sandwiches: cucumber sandwiches (C) and bacon butties (B).

<u>Part 1:</u> Usually, if the weather is sunny (S), the probability that a customer chooses the cucumber sandwich is 4/5. However, if it is rainy (R), the probability that a customer chooses the bacon butty is 3/4.

In Bath, the probability that it rains is 3/5.

- **1)** Draw a tree diagram to summarize the situation.
- **2)** Given that the weather is sunny, what is the probability for a customer to choose a bacon butty?
- **3)** Work out the probability that the weather is sunny and that a customer chooses a bacon butty.
- 4) Explain how you can find the probability that a customer chooses a bacon butty.
- **5)** Are the two events "the customer chooses a bacon butty" and "the weather is sunny" independent? Justify correctly your answer.

<u>Part 2</u>: In May, Jim's best friend comes each day of the month to drink a soda. He chooses a fizzy drink with a probability equal to 0.7. Let assume that his choice of a soda is independent from the choices he made the previous days.

- **6)** What is the probability that Jim's best friend drinks a fizzy drink for 20 days in the month?
- 7) How many fizzy drinks can Jim expect to sell to his friend in May?

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris – Créteil – Versailles

Binôme : Anglais / Mathématiques

D7 - sujet n°7

The first part of this page is a summary that can help you do the exercise.

For any event <i>A</i> and <i>B</i> of the universal set:					
$P(A) = P(A \cap B) + P(A \cap \overline{B})$					
$P(\overline{B}) = 1 - P(B)$					
Conditional probability:					
Let A and B be two events.					
The probability of <i>A</i> given <i>B</i>	$P(A   B) = P(A \cap B)$	provided that $P(B) \neq 0$ .			
is	P(A B) = P(B)	provided that $F(B) \neq 0$ .			
Binomial distribution:					
If the random variable X gives the number of successes in $n$ trials, then X follows the binomial					
distribution with parameters $n$ and $p$ .					
The expectation of a random variable $X$ following a binomial distribution of parameters $n$ and $p$					
is equal to $n \times p$ .					

**EXERCISE**: In a mailbox, messages need to be filtered because it is estimated that 8 % of the received messages are undesirable.

1) Colin decides to use a module designed to filter out undesirable messages.

The module eliminates 96 % of undesirable messages but also eliminates 1 % of welcome messages. Let U be the event "the message is undesirable" and E the event "the message is eliminated".

- a) Draw a probability tree diagram.
- **b)** Prove that the probability of the message being eliminated is equal to 0.086.
- c) What is the probability, if the message is eliminated, that it was welcome (to 3 d.p.)?

**2)** His friend Bob uses a mailbox, without filter. One day, he comes back from holidays and opens his mailbox, which contains 54 mails.

We assume that all the messages are independent since they are coming from different senders. Let *X* be the random variable counting the number of undesirable messages.

- a) Explain why *X* follows a binomial distribution and give its parameters.
- b) What is the probability that Bob gets 7 undesirable messages?
- c) Calculate the expectation of X. What does it represent in the context of the exercise?

# ÉPREUVE SPÉCIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE » Académies de Paris – Créteil – Versailles

Binôme : langue / DNL

D7- Sujet n°8

### COVID VACCINE EFFICIENCY



As of data received through January 3, 2023, the New York State Department of Health is aware of 2 612 690 laboratory-confirmed breakthrough cases of COVID-19 among fully-vaccinated people in New York State (roughly 14 678 034), which corresponds to 17.8% of the population of fully-vaccinated people 5-years or older. And 100 569 hospitalizations with COVID-19 among fully-vaccinated people in New York State, which corresponds to 0.68% of the population of fullyvaccinated people 5-years or older (roughly 14 789 559). To understand the above statistics, it is important to consider that they reflect not only the effectiveness of vaccines, but also changes over time in the intensity of the epidemic, circulating variant strains (such as Omicron), and protective behaviors (e,g, masking and social distancing) against COVID-19, as well as the growing number of people fully-vaccinated in New York State.

From the website "coronavirus.health.ny.gov"

A recent study conducted in the New York State population found that 76% of the population was vaccinated. Approximately 18% of those vaccinated contracted the virus. The ratio of coronavirus cases is estimated to be 30% of the total population in that area. A person is chosen at random from the city's population and the following events are considered:

V: "the person is vaccinated against covid"

G: "the person has contracted covid".

- 1) Explain the figures 17.8% and 0.68% which are mentioned in the previous extract.
- 2) Give the probability of the event G.
- 3) Draw a tree diagram representing the situation which involves the events V and G.
- 4) Determine the probability that the selected person is vaccinated and has contracted covid.
- 5) The selected person is not vaccinated. Compute the probability that she or he has contracted covid.

A pharmaceutical company is conducting a study on covid vaccination in New York city, n inhabitants of the city are randomly interviewed, assuming that this choice is equivalent to carrying out n successive independent draws.

It is assumed that the probability that a randomly selected person in the city is vaccinated against covid is equal to 0.76.

Let us note *X* the random variable equal to the number of vaccinated persons among the *n* questioned.

- 6) What is the probability distribution followed by the random variable *X*?
- 7) Determine the probability that exactly 15 of the 40 people interviewed are vaccinated.
- 8) Determine the probability that at least half of the respondents are vaccinated.
- 9) New York has a recorded average of 73 deaths per day. Let us note *Y* the random variable giving the number of deaths accounted for each day. What is the likelihood of having more than 70 daily deaths?